

Baryon Spectroscopy: New Results and Perspectives

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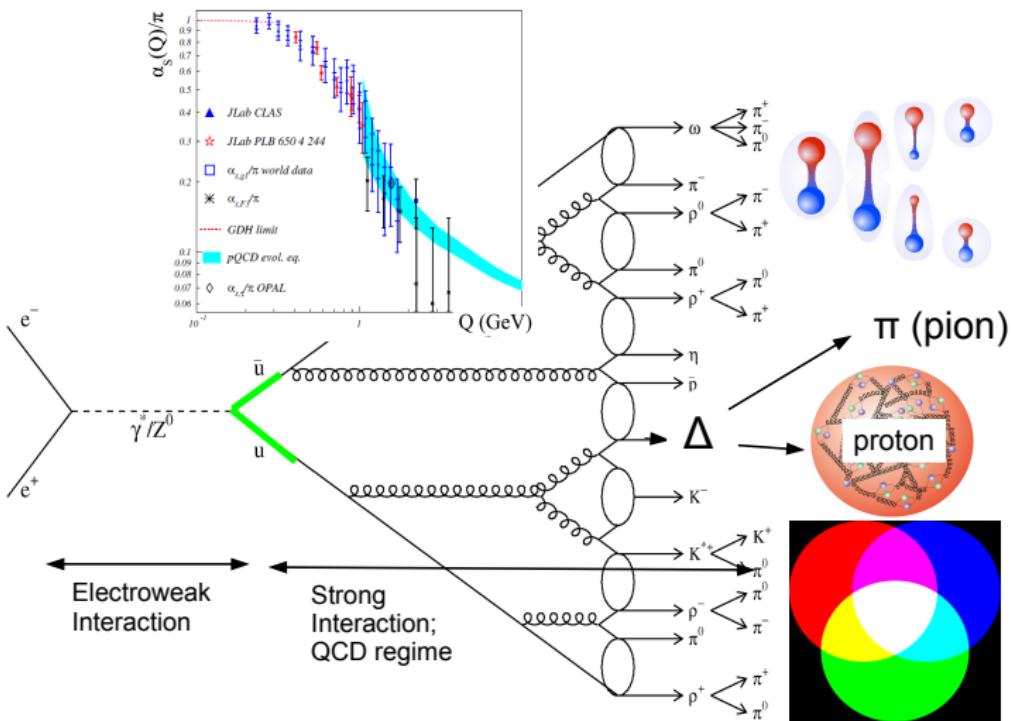
Jülich-Athens-GWU: D. Rönchen, H. Haberzettl, J. Haidenbauer, C. Hanhart, F. Huang, S.
Krewald, U.-G. Meißner, K. Nakayama

GW/INS Data Analysis Centre: W. Briscoe, I. Strakovsky, R. Workman

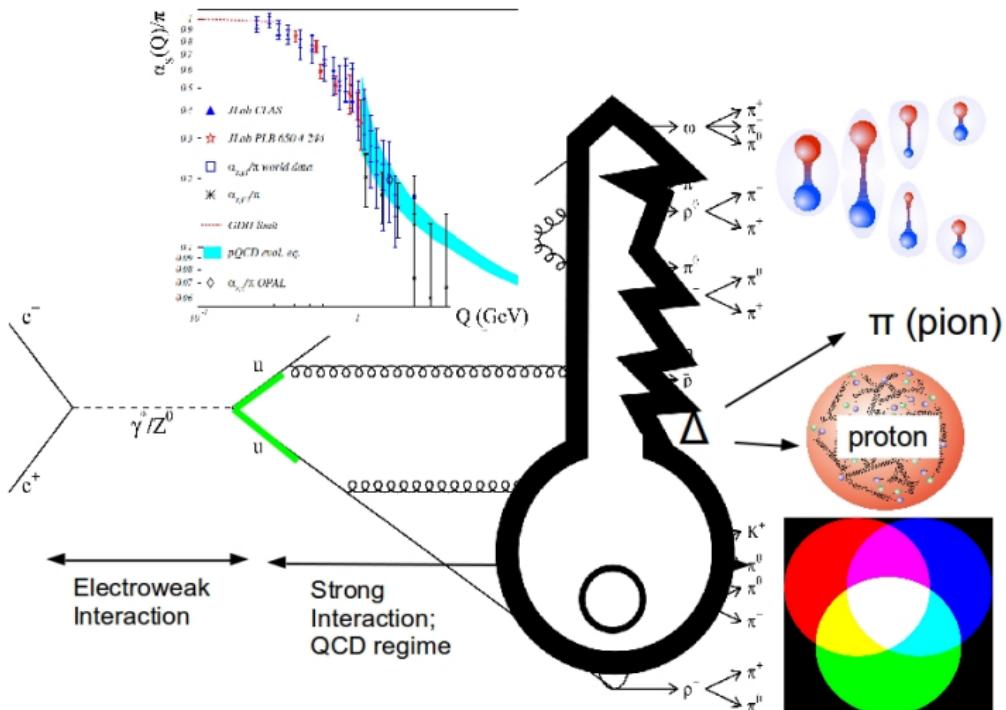
Bonn U.: M. Mai, U.-G. Meißner, A. Rusetsky

Fermilab Theory Seminar, Jan. 29, 2015

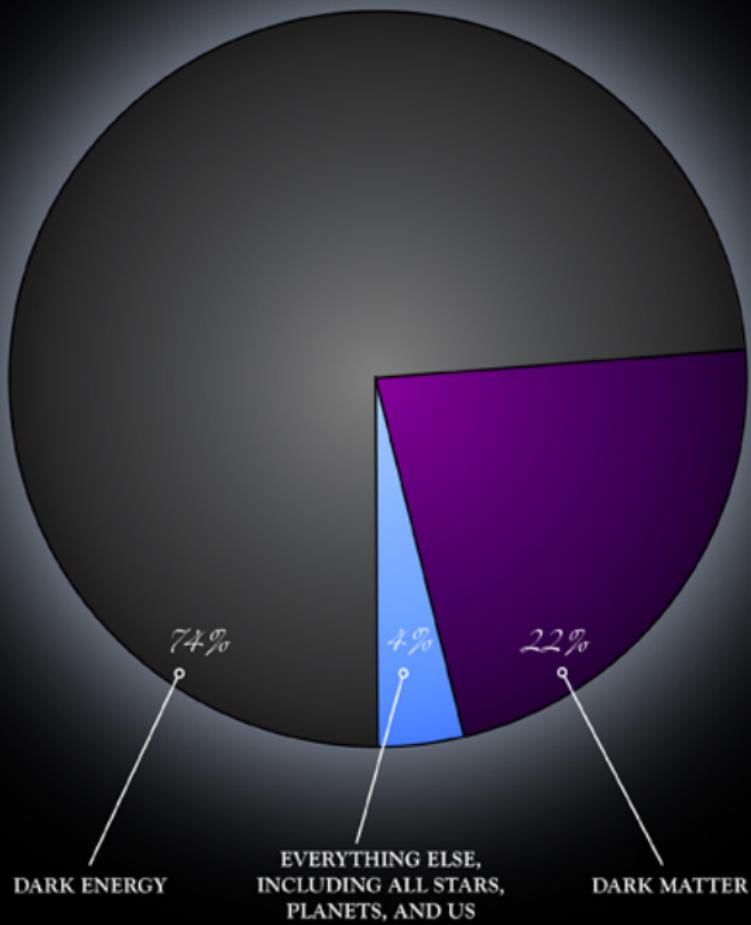
The intermediate energy region...



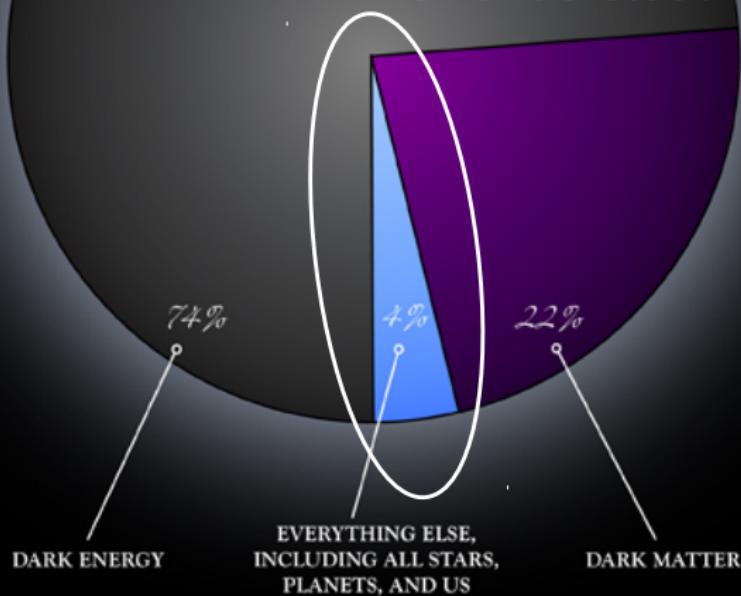
The intermediate energy region...



... provides a key to our understanding of QCD.



Even these 4% not
well understood



Strong interactions

one of the fundamental forces of nature

Interaction between colored **quarks**, mediated by **gluons**

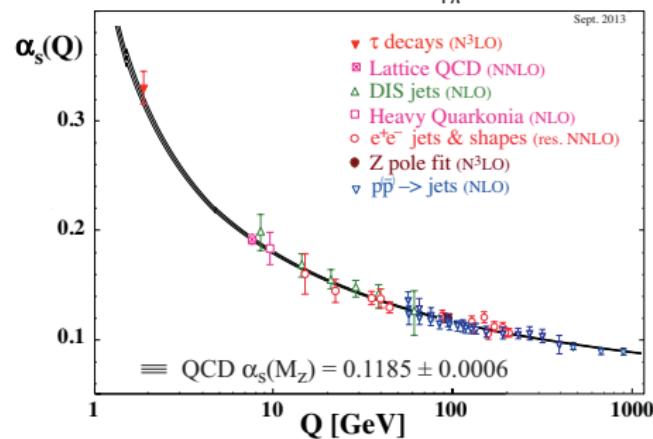
- Quantum Chromodynamics (QCD): gauge field theory of the strong interactions

$$\mathcal{L}_{QCD} = \sum_q \bar{\Psi}_{q,a} (i\partial^\mu \delta_{ab} - g_s \gamma^\mu t^C_{ab} \mathcal{A}_\mu^C - m_q \delta_{ab}) \Psi_{q,b} - \frac{1}{4} F_{\mu\nu}^A F^{A\mu\nu}$$

↑

Running coupling constant $\alpha_s = \frac{g_s^2}{4\pi}$

$\Psi_{q,b}$: quarks
 \mathcal{A}_μ^C : gluon fields
 $F_{\mu\nu}^A$: field tensors



picture by PDG

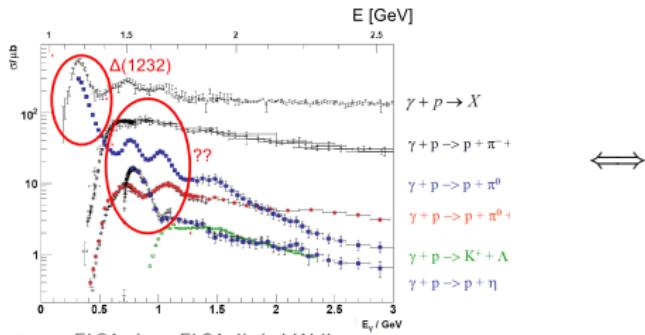
- High energies: asymptotic freedom
- Low energies:
 - no perturbative QCD
 - confinement of quarks in **hadrons**
 - hadrons are color-singlets
- other theoretical approaches (e.g. EFT, DSE, quark models, lattice QCD)

The excited hadron spectrum:

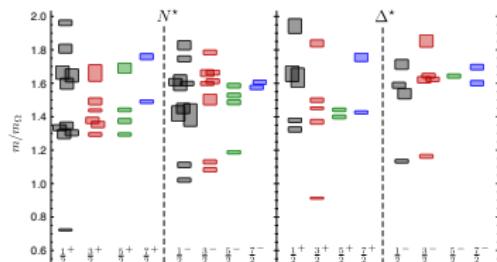
Connection between experiment and QCD in the non-perturbative regime

Excited hadron spectrum: testing ground for theories of the strong force at low and medium energy

Experimental study of hadronic reactions



Theoretical predictions of excited hadrons
e.g. from lattice calculations:



$m_\pi = 396$ MeV [Edwards *et al.*, Phys.Rev. D84 (2011)]

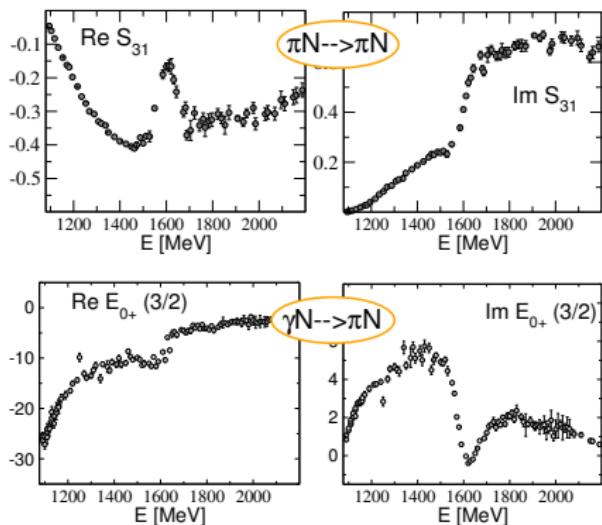
⇒ Partial wave decomposition:

decompose data with respect to a conserved quantum number: J^P

Missing resonance problem

Resonances

$$J^P = 1/2^+, I = 3/2$$



Points: SAID 2006 and CM12

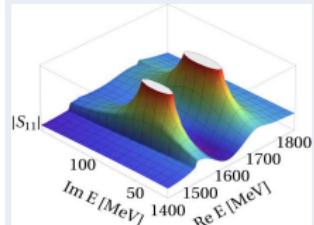
Breit-Wigner parameterization:

$$\mathcal{M}_{ba}^{\text{Res}} = -\frac{g_b g_a}{E^2 - M_{BW}^2 + iE\Gamma_{BW}}$$

- M_{BW}, Γ_{BW} channel dependent
- background? overlapping resonances? thresholds?

Resonances: poles in the T -matrix

- Pole position E_0 is the same in all channels
- thresholds: branch points



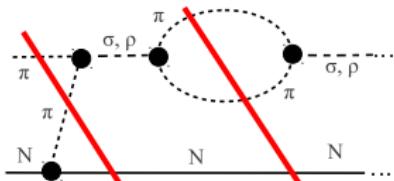
$\text{Re}(E_0)$ = "mass"
 $-2\text{Im}(E_0)$ = "width"
 residues \rightarrow branching ratios

The scattering matrix

$$S = \mathbf{1} + iT$$

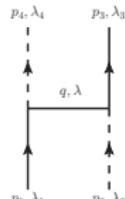
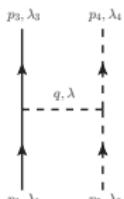
Unitarity: $SS^\dagger = 1 \Leftrightarrow -i(T - T^\dagger) = T T^\dagger$

- 3-body unitarity:
discontinuities from t -channel exchanges
→ Meson exchange from requirements of
the S -matrix [Aaron, Almado, Young, Phys. Rev. 174, 2022 (1968)]



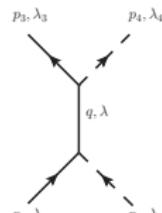
Other cuts

- to approximate left-hand cut → Baryon u -channel exchange
- σ, ρ exchanges from crossing plus analytic continuation.

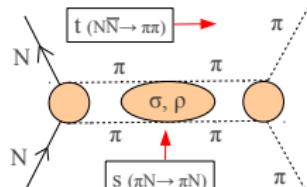


$$\vec{q} = \vec{p}_1 - \vec{p}_3$$

$$\vec{q} = \vec{q}_1 - \vec{p}_4$$



$$\vec{q} = \vec{p}_1 + \vec{p}_2$$

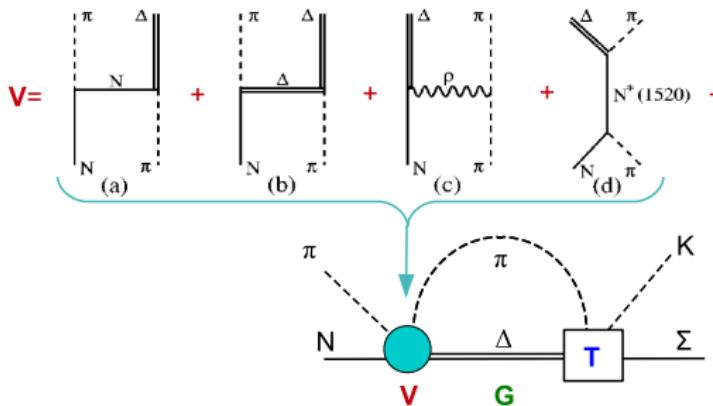


A dynamical coupled-channel approach: the hadronic Jülich model

Dynamical coupled-channels (DCC): simultaneous analysis of different reactions

The scattering equation in partial wave basis

$$\langle L'S'p' | \textcolor{blue}{T}_{\mu\nu}^{IJ} | LSp \rangle = \langle L'S'p' | \textcolor{red}{V}_{\mu\nu}^{IJ} | LSp \rangle + \sum_{\gamma, L''S''} \int_0^\infty q^2 dq \langle L'S'p' | \textcolor{red}{V}_{\mu\gamma}^{IJ} | L''S''q \rangle \frac{1}{E - E_\gamma(q) + i\epsilon} \langle L''S''q | \textcolor{blue}{T}_{\gamma\nu}^{IJ} | LSp \rangle$$



- potentials $\textcolor{red}{V}$ constructed from effective \mathcal{L}
- s -channel diagrams: T^P
genuine resonance states
- t - and u -channel: T^{NP}
dynamical generation of poles
partial waves strongly correlated

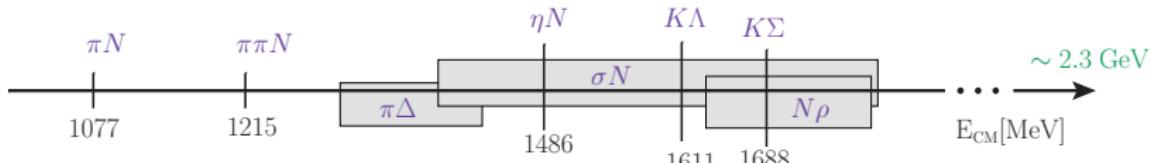
A dynamical coupled-channel approach: the hadronic Jülich model

Dynamical coupled-channels (DCC): **simultaneous** analysis of different reactions

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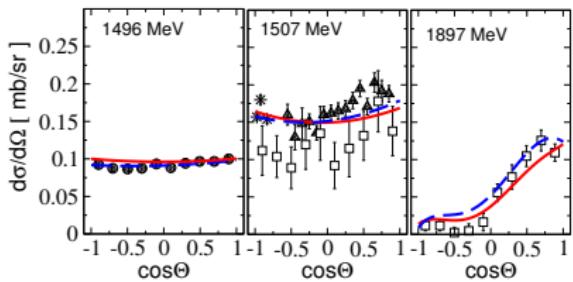
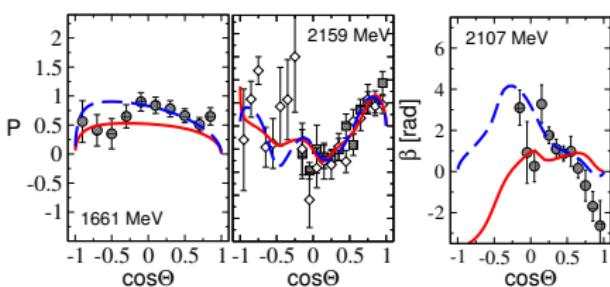
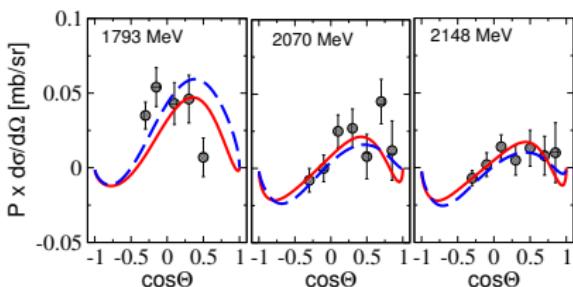
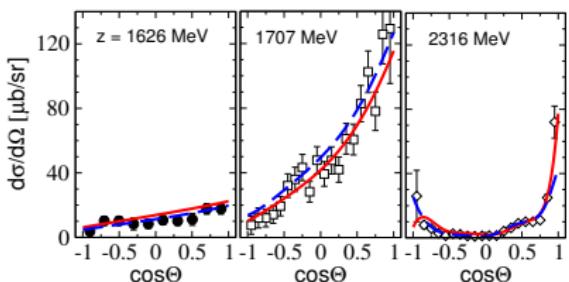
- **Analyticity** is respected (correct structure of branch points and cuts)
 ↪ reliable extraction of resonance parameters
- **Unitarity**
- $J \leq 9/2$



$\pi N \rightarrow \eta N, K\Lambda$

selected results

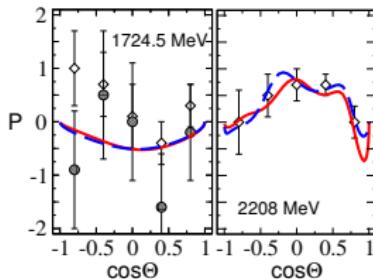
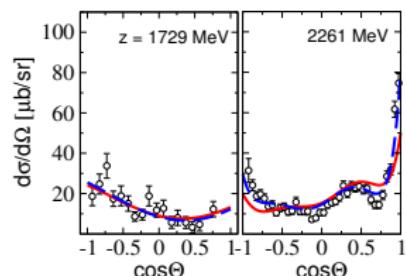
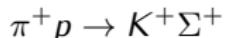
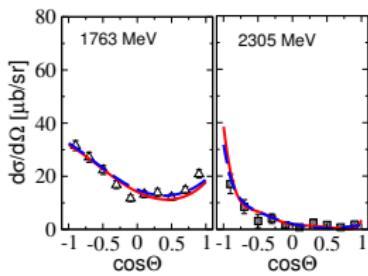
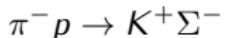
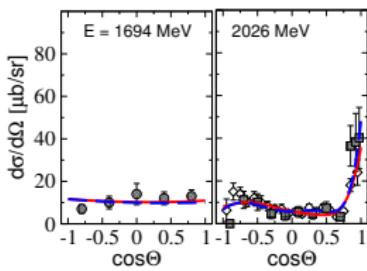
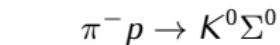
Eur.Phys.J. A49 (2013) 44, Nucl.Phys. A851 (2011) 58-98

 $\pi^- p \rightarrow \eta N$  $\pi^- p \rightarrow K^0 \Lambda$ 

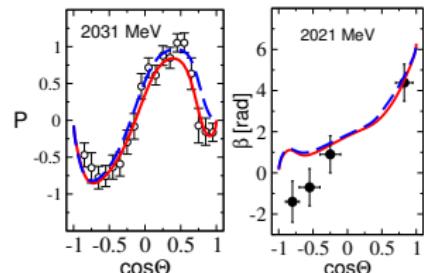
Full results



selected results

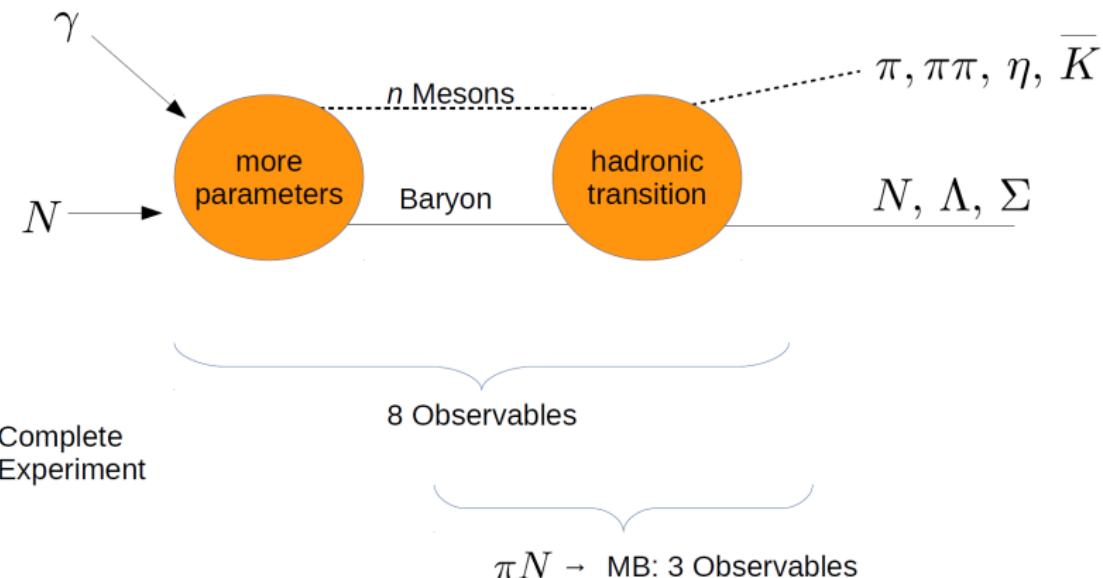


No polarization data!



Full results

Photon-induced vs. Pion-induced reactions



Hadronic transition ALONE fix pole positions and strong branching ratios
→ Principal point of comparison with lattice QCD.

Coupled-channels: Any problematic data in MB → MB will cause problems in photoproduction analysis.

PHYSICS OPPORTUNITIES WITH MESON BEAMS

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Megumi Naruki³, Igor I. Strakovsky¹, Eric S. Swanson⁴

¹ The George Washington University, Washington, DC 20052, USA

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³ Kyoto University, Kyoto 606-8502, Japan

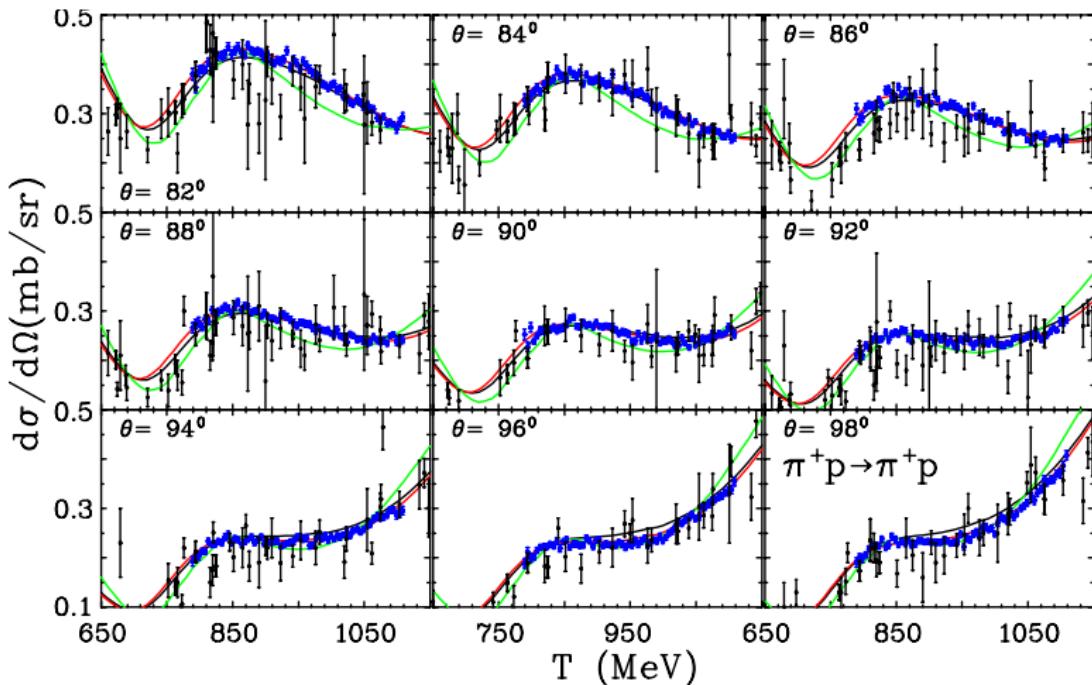
⁴ University of Pittsburgh, Pittsburgh, PA 15260, USA

Abstract

High statistics data from measurements with meson beams can revolutionize baryon and meson spectroscopy and other related areas of hadron physics. Measurements are needed with good angular and energy coverage for a wide range of reactions using both polarized and unpolarized targets. Such data are critically needed to advance the work carried out worldwide over the past two decades at electromagnetic facilities. Many millions of dollars have been invested at such facilities only to measure extremely precise meson photoproduction data that are today being analyzed with hadronic input of poor quality. An opportunity exists to remedy this problem by constructing a meson beam facility as part of the coming JLab EIC complex.

Improvement in Modern Experimental Facilities: $\pi N \rightarrow \pi N$

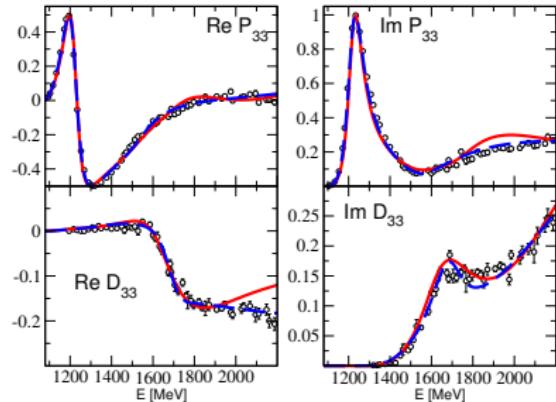
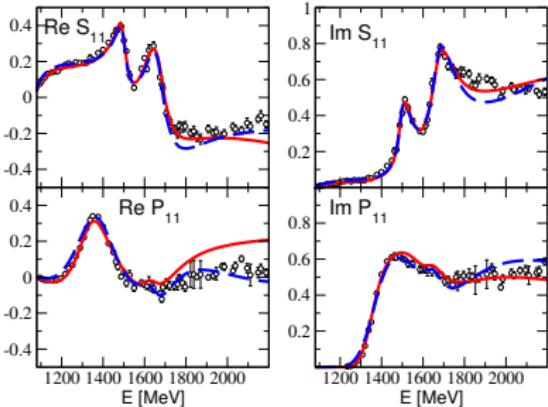
EPECUR & GWU/SAID, Alekseev *et al.*, arXiv:1410.6418



Black: WI08 prediction; Red: WI14 fit; green: KA84.

$\pi N \rightarrow \pi N$ partial wave amplitudes

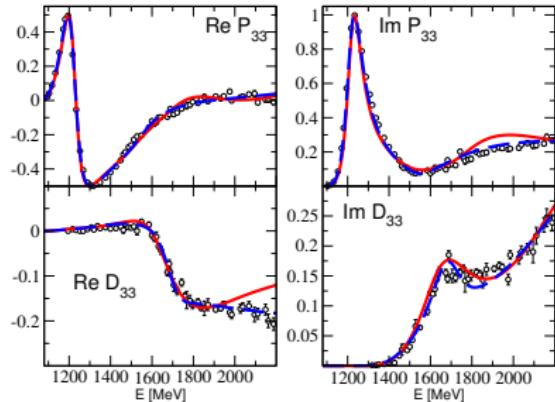
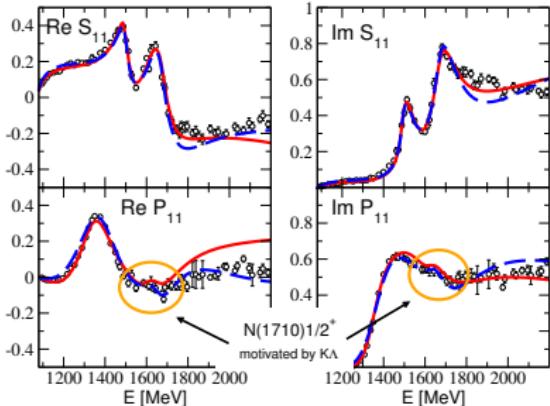
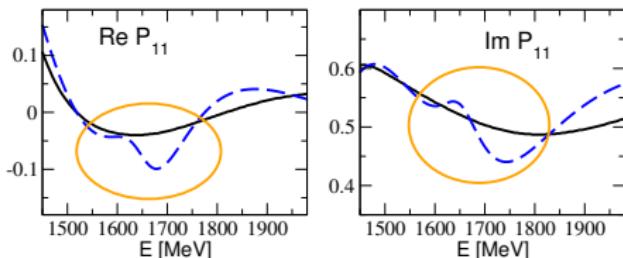
Coupled-channels at work



- Notation: L_{2I2J}
- Input to fit: energy-dependent partial wave analysis, GWU/SAID 2006 up to $J = 9/2$ ($\sim H_{39}$)

$\pi N \rightarrow \pi N$ partial wave amplitudes

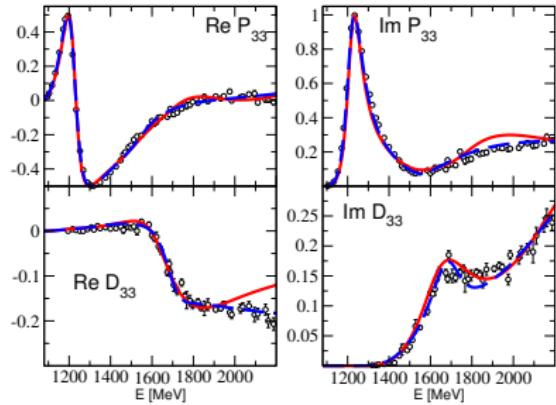
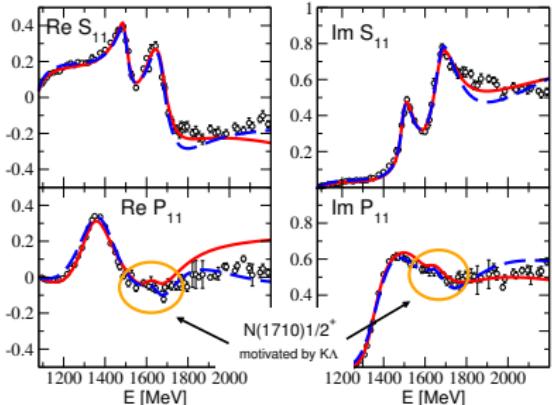
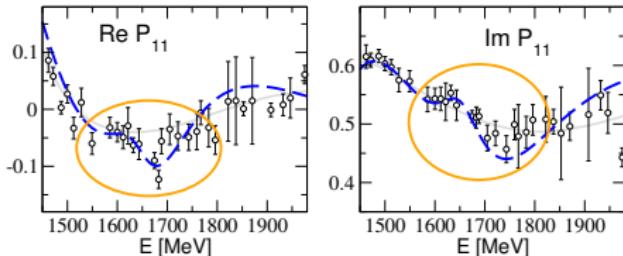
Coupled-channels at work

Genuine $N(1710) 1/2^+$:

- Black line: input to fit (energy-dependent solution)
- Inclusion of $P_{11}(1710)$ necessary to improve $K\Lambda$
- Coupled-channels essential (Fit to πN observables)

$\pi N \rightarrow \pi N$ partial wave amplitudes

Coupled-channels at work

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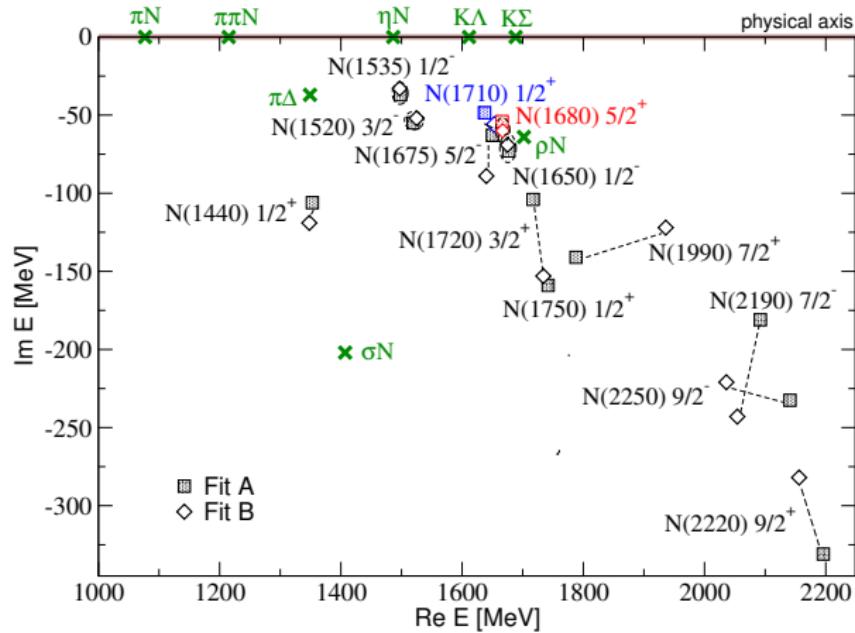
Resonance content: $I=1/2$

Pole search on the 2nd sheet of the scattering matrix $T_{\mu\nu}$

Resonance parameter:

- "mass" = $Re(E_0)$
- "width" = $-2Im(E_0)$
- Residues → branching ratios

E_0 : pole position



Numbers

x: branch points

Notation: $N(\text{"name"}) J^{parity}$

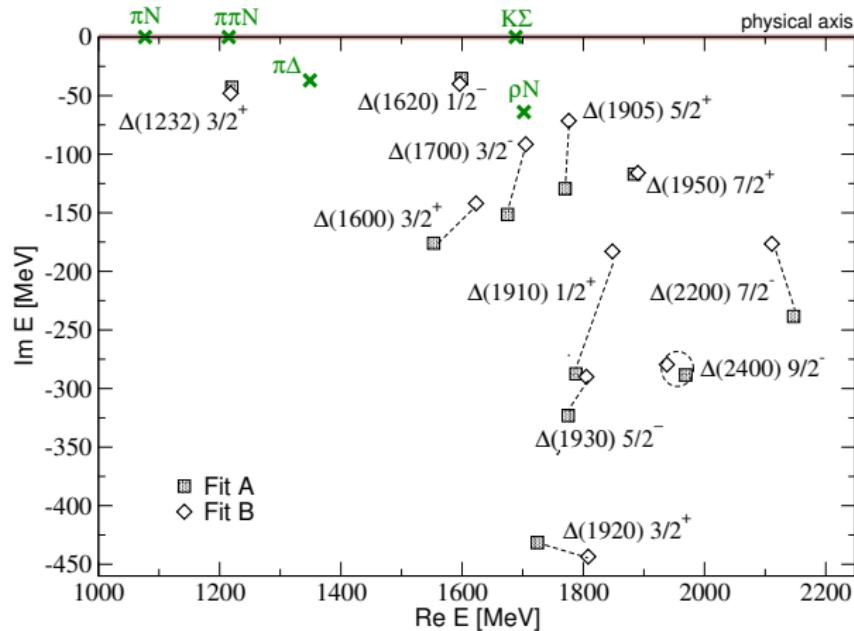
Resonance content: $I=3/2$

Pole search on the 2nd sheet of the scattering matrix $T_{\mu\nu}$

Resonance parameter:

- "mass" = $Re(E_0)$
- "width" = $-2Im(E_0)$
- Residues → branching ratios

E_0 : pole position



Numbers

x: branch points

Notation: $\Delta(\text{"name"}) J^{parity}$

Photoproduction of pseudoscalar mesons

[D. Rönchen, M.D., et. al., Eur. Phys. J. A (2014) 50: 101]

- Photocouplings of resonances
- high precision data from ELSA, MAMI, JLab... → resolve questionable/find new states

Photoproduction amplitude of pseudoscalar mesons:

$$\hat{\mathcal{M}} = F_1 \vec{\sigma} \cdot \vec{\epsilon} + iF_2 \vec{\epsilon} \cdot (\hat{k} \times \hat{q}) + F_3 \vec{\sigma} \cdot \hat{k} \hat{q} \cdot \vec{\epsilon} + F_4 \vec{\sigma} \cdot \hat{q} \hat{q} \cdot \vec{\epsilon}$$

\vec{q} : meson momentum

\vec{k} ($\vec{\epsilon}$): photon momentum
(polarization)

F_i : complex functions of the scattering angle

⇒ 16 polarization observables:

asymmetries composed of beam, target and/or recoil polarization measurements

⇒ 8 carefully selected observables: *complete experiment*

→ Caveat: in reality more observables needed (data uncertainties)

Focus of the present analysis:

extraction of electromagnetic resonance parameters

⇒ flexible, “model-independent” parameterization of photo excitation

- Advantage: easy to implement, analyze large amounts of data
- Disadvantage: no information on microscopic reaction dynamics

⇒ intermediate step
towards a combined DCC
analysis of pion- and
photon-induced reactions

Data base of $\gamma p \rightarrow \pi^0 p, \pi^+ n$

| | Fit 1 | Fit 2 |
|---------------------------|---|-----------|
| Line style | — · — · — | — |
| # of data | 21,627 | 23,518 |
| Excluded data | $\pi^0 p$: $E > 2.33$ GeV and $\theta < 40^\circ$ for $E > 2.05$ GeV $\pi^+ n$: $E > 2.26$ GeV and $\theta < 9^\circ$ for $E > 1.60$ GeV | |
| $ds/d\Omega, P, T$ | included | included |
| Σ | included | included |
| | (CLAS 2013 predicted) | |
| $\Delta\sigma_{31}, G, H$ | predicted | included |
| $E, F, C_{x'L}, C_{z'L}$ | predicted | predicted |

Complete experiment

one possibility:

$\sigma, \Sigma, T, P, E, G, C_x, C_z$

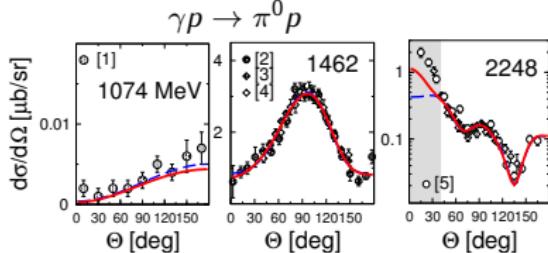
⇒ Influence of new double polarization observables on resonance parameters

Minimization using parallelized code on JUROPA (FZ Jülich)

Single polarization observables

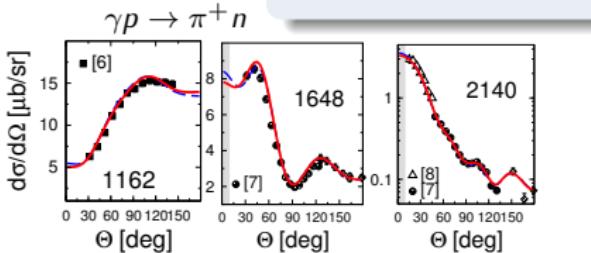
selected results

$d\sigma/d\Omega$



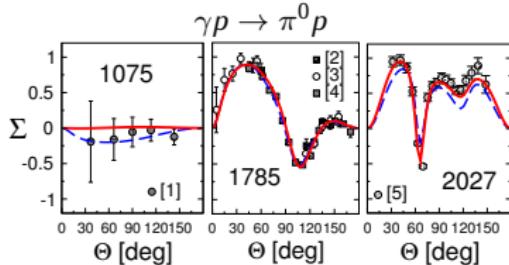
- [1] Schmidt 2001 (MAMI)
- [2] Beck 2006 (MAMI)
- [3] Dugger 2007 (JLab)
- [4] Bartholomy 2005 (ELSA)
- [5] Credé 2011 (ELSA)

Polarization:

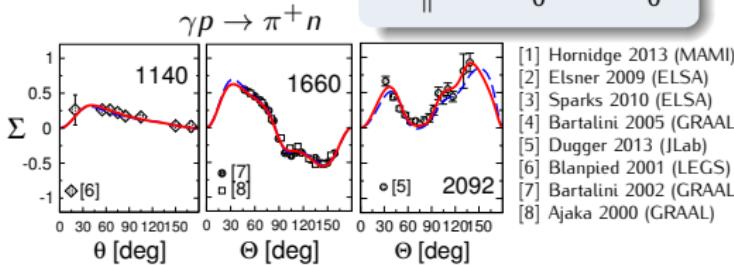


- [6] Ahrens 2004 (MAMI)
- [7] Dugger 2009 (JLab)
- [8] Buschhorn 1966

Beam asymmetry Σ



Polarization:

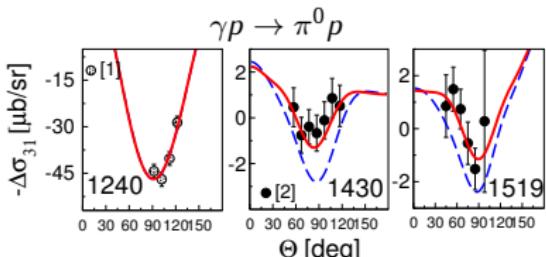


- [1] Hornidge 2013 (MAMI)
- [2] Elsner 2009 (ELSA)
- [3] Sparks 2010 (ELSA)
- [4] Bartalini 2005 (GRAAL)
- [5] Dugger 2013 (JLab)
- [6] Blanpied 2001 (LEGS)
- [7] Bartalini 2002 (GRAAL)
- [8] Ajaka 2000 (GRAAL)

Double polarization observables

- $\Delta\sigma_{31}$

$$\Delta\sigma_{31} = -2 \frac{d\sigma}{d\Omega} E$$

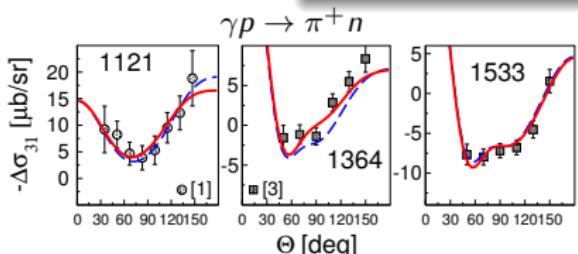


[1] Ahrens *et al.* 2004 (MAMI) [2] Ahrens *et al.* 2002 (MAMI)

selected results

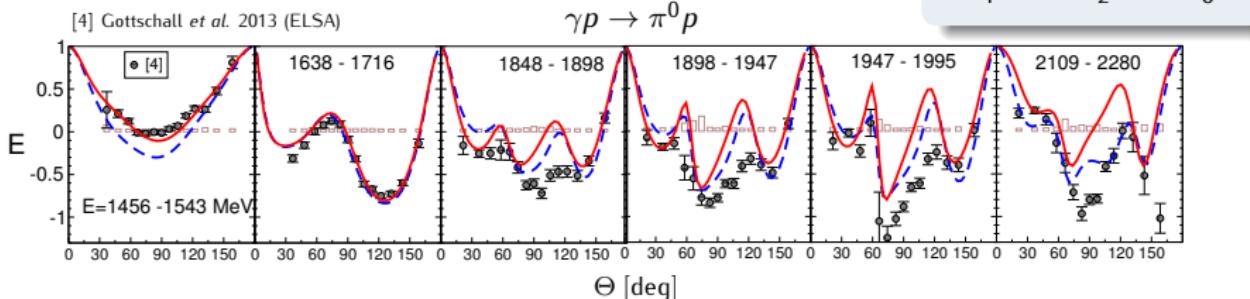
dashed blue line: prediction
solid red line: fit

| Beam | Target | Recoil |
|------|--------|--------|
| +1 | $-z$ | 0 |
| -1 | $-z$ | 0 |



[3] Ahrens *et al.* 2006 (MAMI)

- Predictions of E

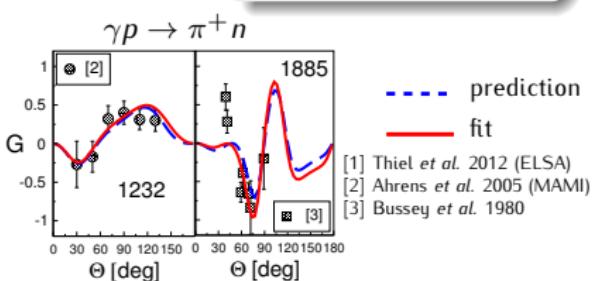
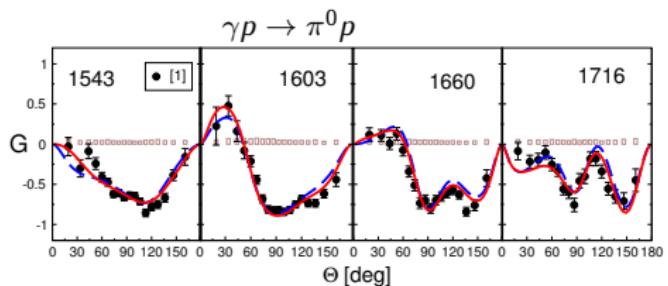
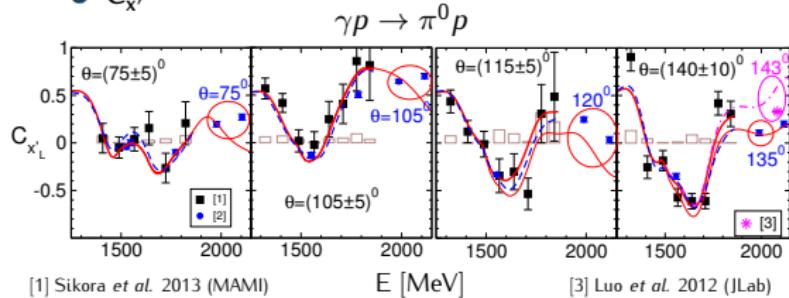


| Beam | Target | Recoil |
|------|--------|--------|
| +1 | $-z$ | 0 |
| -1 | $-z$ | 0 |

Double polarization observables

selected results

• G

• $C_{x'}$ 

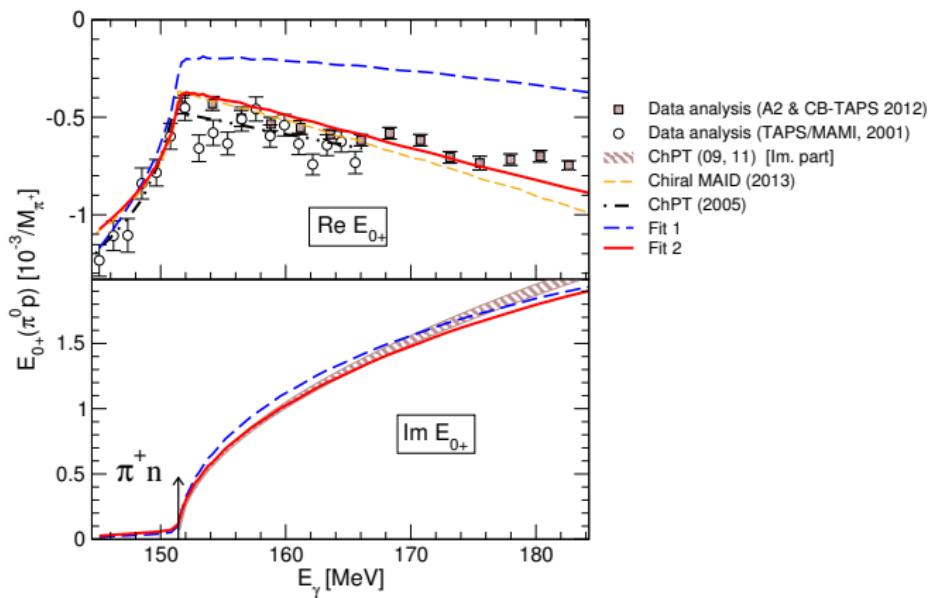
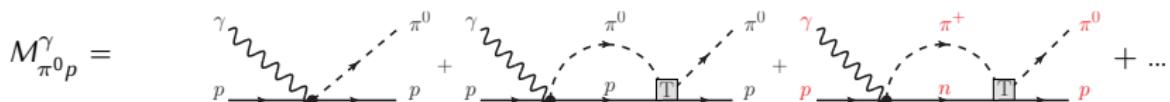
[1] Sikora et al. 2013 (MAMI)

[2] Wijesooriya et al. 2002 (JLab)

| Beam | Target | Recoil |
|--------------|--------|--------|
| \perp' | $-z$ | 0 |
| \parallel' | $-z$ | 0 |

Predictions of

- blue dashed line: fit 1, angle averaged
- red solid line: fit 2, angle averaged
- red solid line: fit 2, not angle averaged
- magenta dashed line: fit 2, not angle averaged

Multipoles: $E_{0+}(\pi^0 p)$ at threshold

Different π & N
masses
→ cusp effect

Photocouplings at the pole

selected results

$$\tilde{A}_{pole}^h = A_{pole}^h e^{i\vartheta^h}$$

$h = 1/2, 3/2$

$$\tilde{A}_{pole}^h = I_F \sqrt{\frac{q_p}{k_p} \frac{2\pi (2J+1) E_0}{m_N r_{\pi N}}} \text{Res } A_{L\pm}^h$$

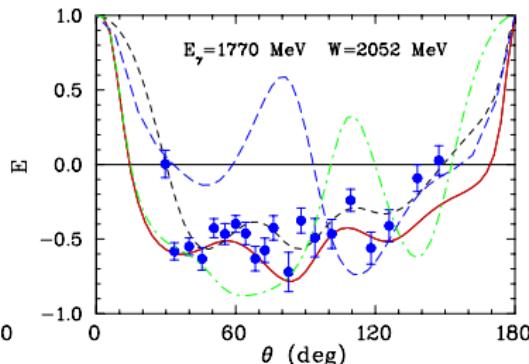
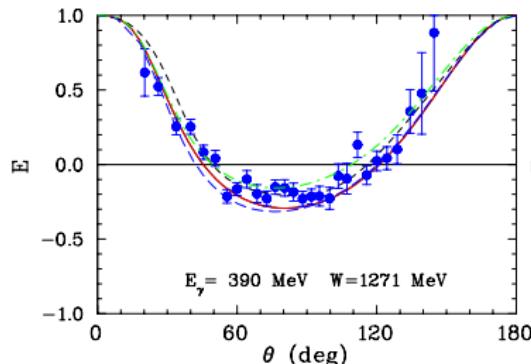
 I_F : isospin factor $q_p (k_p)$: meson (photon) momentum at the pole $J = L \pm 1/2$ total angular momentum E_0 : pole position $r_{\pi N}$: elastic πN residue

| | $A_{pole}^{1/2}$ | | $\vartheta^{1/2}$ | | $A_{pole}^{3/2}$ | | $\vartheta^{3/2}$ | |
|------------------------|--------------------------------|-------------------|-------------------|-----------------|--------------------------------|------------------|-------------------|---------------------|
| | $[10^{-3} \text{ GeV}^{-1/2}]$ | | [deg] | | $[10^{-3} \text{ GeV}^{-1/2}]$ | | [deg] | |
| | fit→ | 1 | 2 | 1 | 2 | 1 | 2 | 1 |
| $N(1710) \ 1/2^+$ | 15 | 28^{+9}_{-2} | 13 | 77^{+20}_{-9} | | | | |
| $\Delta(1232) \ 3/2^+$ | -116 | -114^{+10}_{-3} | -27 | -27^{+4}_{-2} | -231 | -229^{+3}_{-4} | -15 | $-15^{+0.3}_{-0.4}$ |

Fit 1: only **single** polarization observables includedFit 2: also **double** polarization observables included

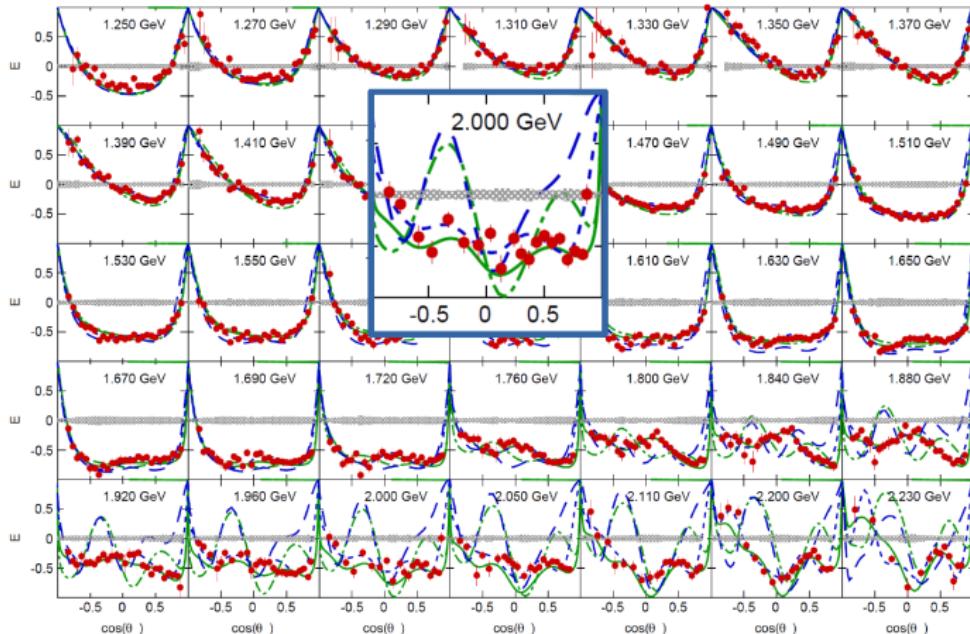
First FROST results: E in $\vec{\gamma}\vec{p} \rightarrow \pi^+ n$

S. Strauch et al., in preparation



- Blue dashed: Prediction BnGa Eur. Phys. J. A 48, 15 (2012)
- Green dashed-dotted: Prediction MAID07 Eur. Phys. J. A 34, 69 (2007)
- Red solid: Refit DAC at GWU (SAID)
- Black short-dashed; Refit Jülich

Refits (Jülich & GWU/SAID)



Data: CLAS (USC/Strauch et al.); preliminary

SAID analysis (prediction and re-analysis)

Jülich Athens Washington (prediction and re-analysis)

→ Significant changes of helicity amplitudes $A^{1/2}$, $A^{3/2}$

Impact on resonance parameters

- $A [10^{-3} \text{ GeV}^{-1/2}]$; $\vartheta [\text{deg}]$.
- Pole values (P) or Breit-Wigner Parameters (BW).
- ";" separates before and after new FROST E -measurement.

| | | | $A_{pole}^{1/2}$ | $\vartheta^{1/2}$ | $A_{pole}^{3/2}$ | $\vartheta^{3/2}$ |
|-----------|---------|--|---|-------------------------------|------------------|-------------------|
| $N(1535)$ | $1/2^-$ | | | | | |
| SAID | BW | | $120 \pm 10; 128 \pm 4$ | | | |
| Jülich | P | | $49; 50$ | $-46; -45$ | | |
| BnGa | P | | 116 ± 10 | 7 ± 6 | | |
| ANL-Osaka | P | | 161 | 9 | | |
| WTS | P | | 77 ± 5 | 4 | | |
| PDG | BW | | 90 ± 30 | | | |
| $N(1650)$ | $1/2^-$ | | | | | |
| SAID | BW | | $60 \pm 30; 55 \pm 30$ | | | |
| Jülich | P | | $26; 23$ | $-58; -29$ | | |
| BnGa | P | | 33 ± 7 | -9 ± 15 | | |
| ANL-Osaka | P | | 40 | -44 | | |
| WTS | P | | 35 ± 3 | -16 | | |
| PDG | BW | | 53 ± 16 | | | |

Impact on resonance parameters

| | | | $A_{pole}^{1/2}$ | $\vartheta^{1/2}$ | $A_{pole}^{3/2}$ | $\vartheta^{3/2}$ |
|---|----|--|---------------------------------|-------------------|-------------------------------|-------------------|
| $N(1440) \frac{1}{2}^+$ | | | | | | |
| SAID | BW | | -60 \pm 5; -56 \pm 1 | | | |
| Jülich | P | | -52; -54 | -51; -43 | | |
| BnGa | P | | -44 \pm 7 | -38 \pm 5 | | |
| ANL-Osaka | P | | 49 | -10 | | |
| WTS | P | | -66 \pm 5 | -38 | | |
| PDG | BW | | -60 \pm 4 | | | |
| $N(1675) \frac{5}{2}^-$ | | | | | | |
| SAID | BW | | 10 \pm 3; 13 \pm 1 | | 16 \pm 4; 16 \pm 1 | |
| Jülich | P | | 28 ; 22 | 40 ; 38 | 63 ; 36 | -19; -41 |
| BnGa | P | | 24 \pm 3 | -16 \pm 5 | 26 \pm 8 | -19 \pm 6 |
| ANL-Osaka | P | | 5 | -22 | 33 | -23 |
| PDG | BW | | 19 \pm 8 | | 15 \pm 9 | |

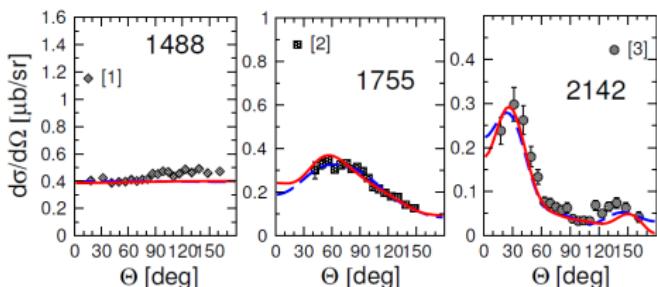
Fit results $\gamma p \rightarrow \eta p$

selected results, preliminary

T, F not included

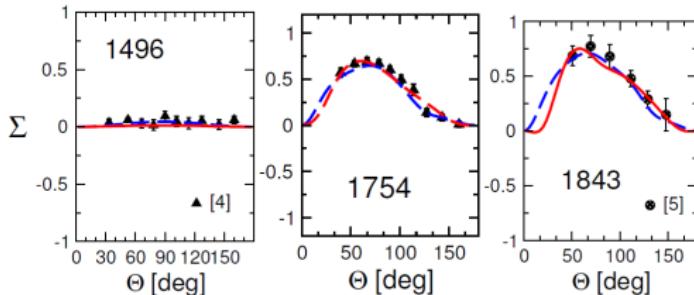
T, F included

Differential cross section



[1] McNicoll *et al.* 2010 (MAMI), [2] Williams *et al.* 2009 (JLab), [3] Credé *et al.* 2009 (ELSA)

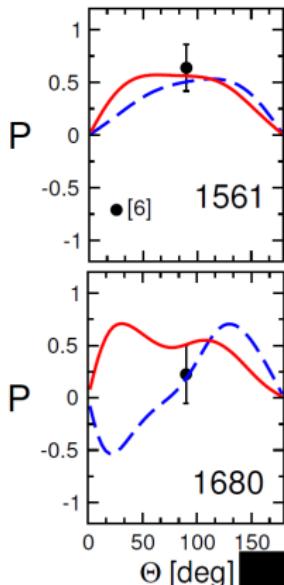
Beam asymmetry



[4] Bartalini *et al.* 2007 (GRAAL), [5] Elsner *et al.* 2007 (ELSA)

Recoil polarization

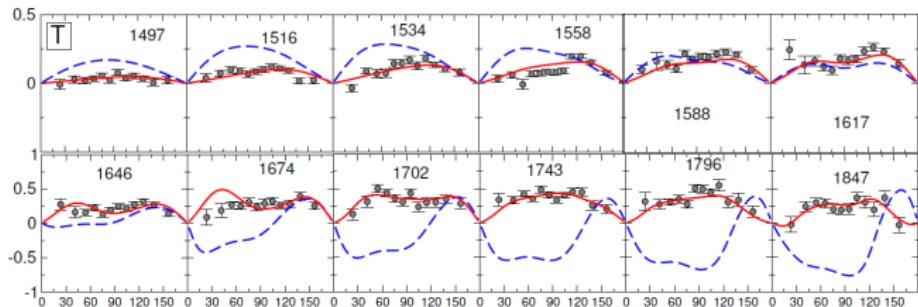
- only 7 data points in total -



[6] Heusch *et al.* (Caltech),
PRL25, 1381 (1970)

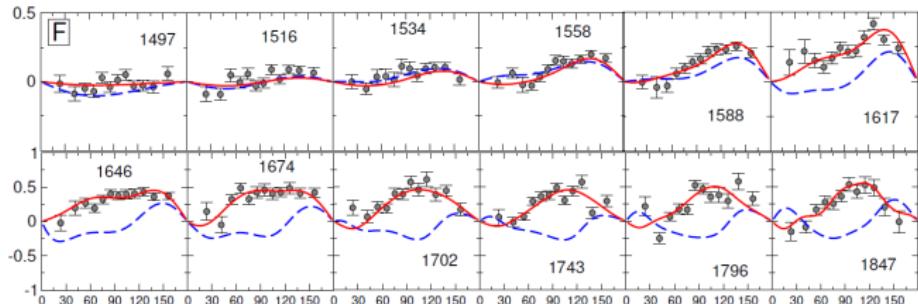
T and F in $\gamma p \rightarrow \eta p$ (MAMI)

preliminary

Data: Akondi *et al.* (A2 at MAMI) PRL 113, 102001 (2014)

— prediction
— fit

| Beam | Target | Recoil |
|------|--------|--------|
| 0 | +y | 0 |
| 0 | -y | 0 |



| Beam | Target | Recoil |
|------|--------|--------|
| +1 | +x | 0 |
| -1 | +x | 0 |

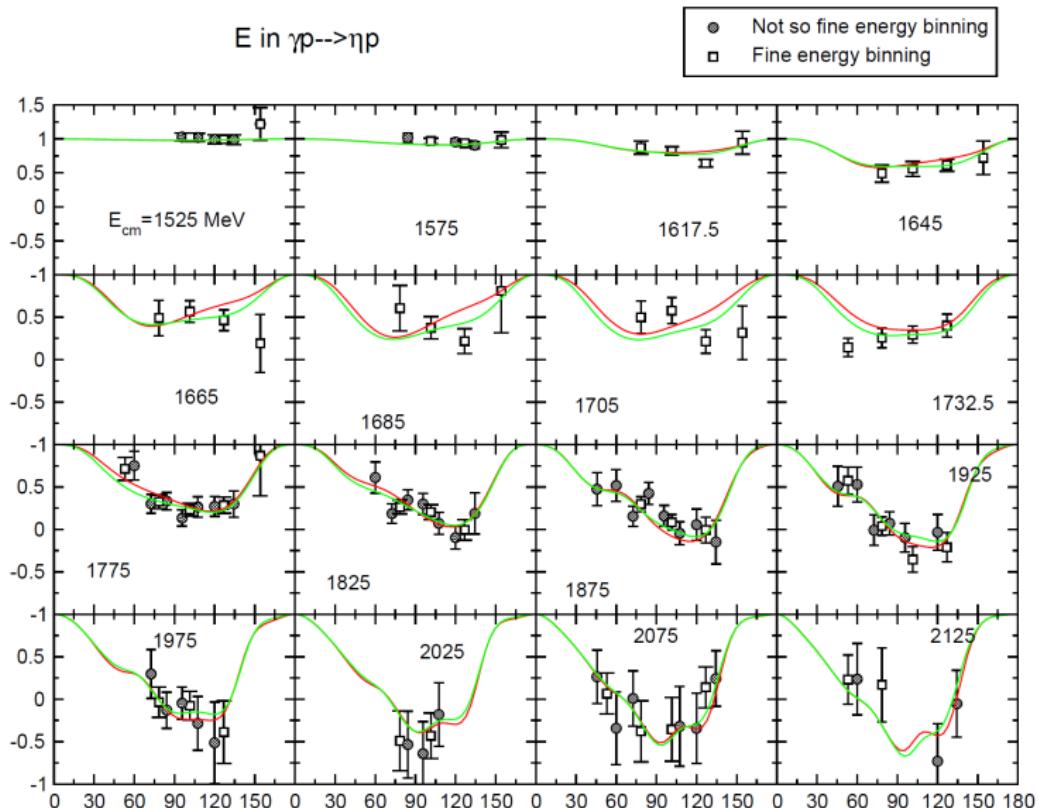
Future data challenges: FROST, HD-ICE at JLab

| | σ | Σ | T | P | E | F | G | H | T_x | T_z | L_x | L_z | O_x | O_z | C_x | C_z |
|--|----------|----------|---|---|---|---|---|---|-------|-------|-------|-------|-------|-------|-------|-------|
| Proton target | | | | | | | | | | | | | | | | |
| $p\pi^0$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | | | | | | |
| $n\pi^+$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | | | | | | |
| $p\eta$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | | | | | | |
| $p\eta'$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | | | | | | |
| $K^+\Lambda$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $K^+\Sigma^0$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $K^0\Sigma^+$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | | | | | | |
| "Neutron" target | | | | | | | | | | | | | | | | |
| $p\pi^-$ | ✓ | ✓ | ✓ | | ✓ | ✓ | ✓ | ✓ | | | | | | | | |
| $K^+\Sigma^-$ | ✓ | ✓ | ✓ | | ✓ | ✓ | ✓ | ✓ | | | | | | | | |
| $K^0\Lambda$ | ✓ | ✓ | ✓ | | ✓ | ✓ | ✓ | ✓ | | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $K^0\Sigma^0$ | ✓ | ✓ | ✓ | | ✓ | ✓ | ✓ | ✓ | | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| ✓ - published ✓ - acquired | | | | | | | | | | | | | | | | |

FROST E in $\gamma p \rightarrow \eta p$

Preliminary data thanks to M. Dugger and I. Senderovich

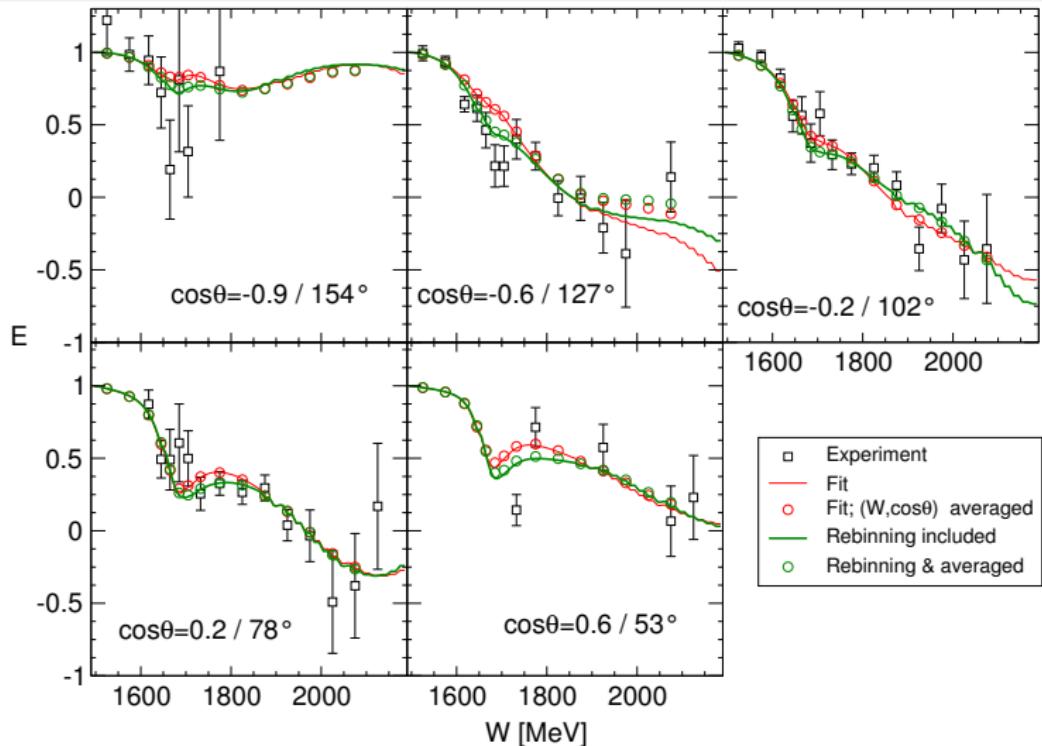
preliminary



FROST E in $\gamma p \rightarrow \eta p$: Excitation function

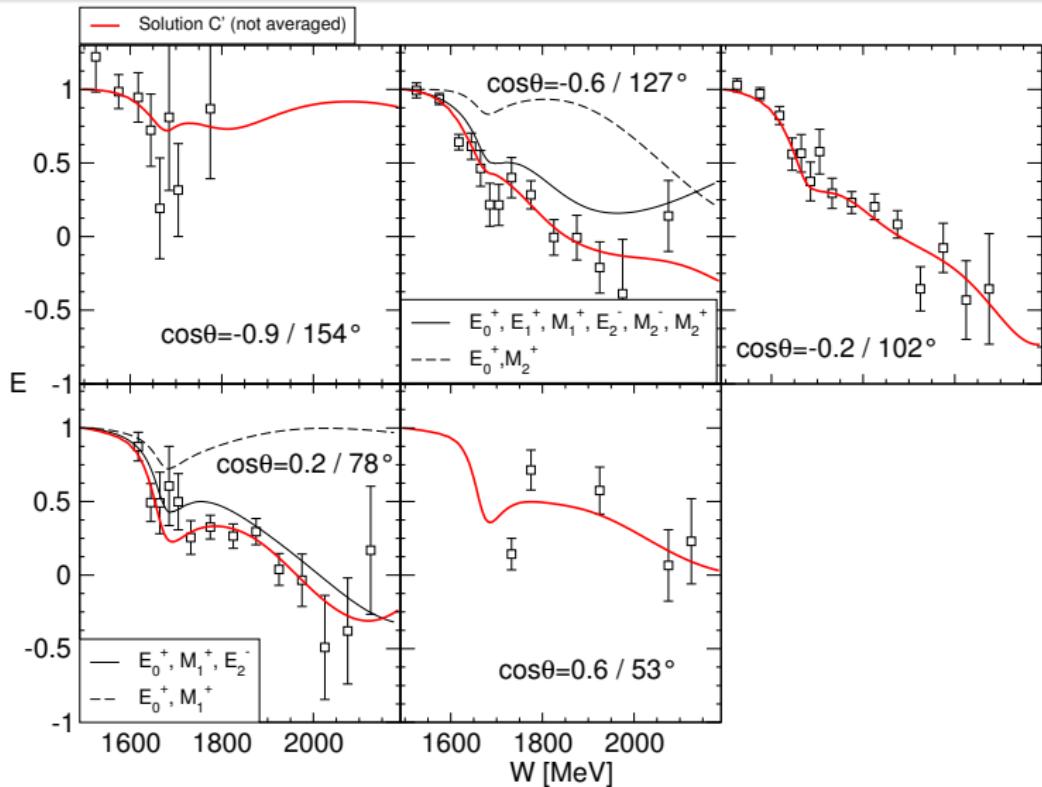
Preliminary data thanks to M. Dugger and I. Senderovich

preliminary

NO additional structure at $W = 1.68$ GeV \rightarrow interferences & $K\Sigma$ threshold.

Structure at $W = 1.68$ GeV: Conventional Resonances plus $K\Sigma$ Cusp

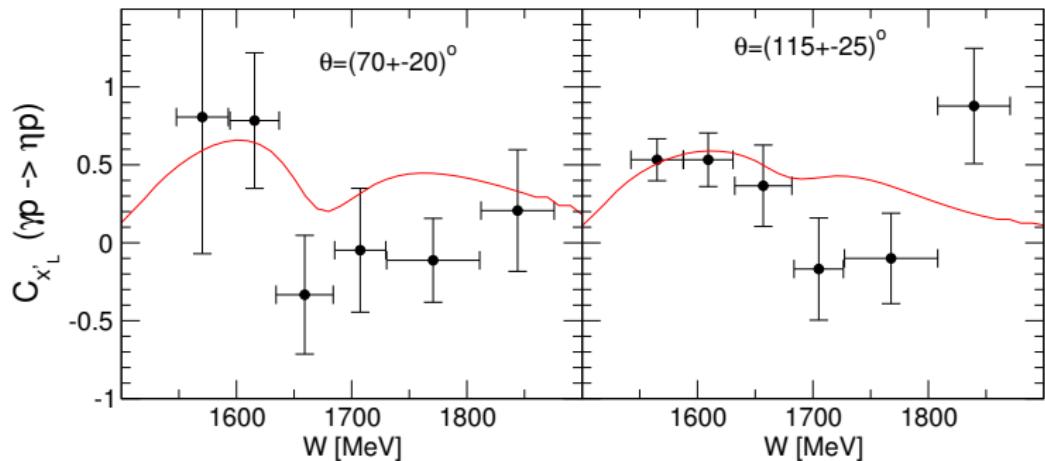
Preliminary data thanks to M. Dugger and I. Senderovich



MAMI $C_{x'}$ in $\gamma p \rightarrow \eta p$

Preliminary data thanks to D. Watts and M. Sikora

preliminary



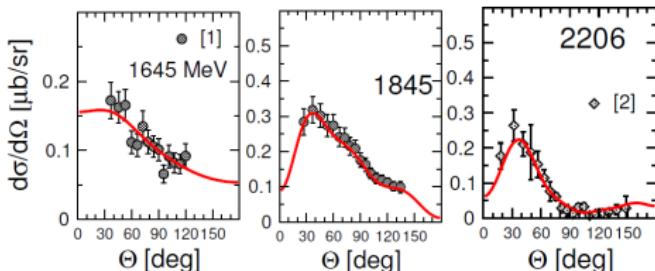
(Data not included in fit)

First results in $\gamma p \rightarrow K\Lambda$

very preliminary

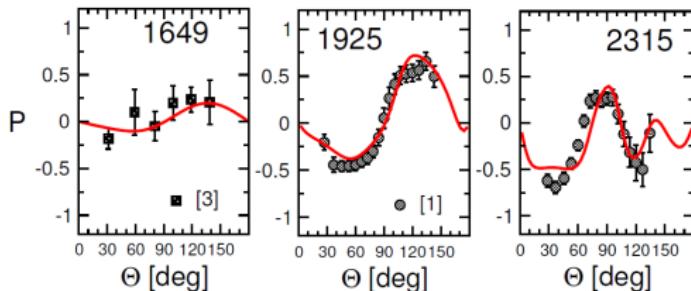
simultaneous fit of $\gamma p \rightarrow \pi^0 p, \pi^+ n, \eta p, K^+ \Lambda$
and $\pi N \rightarrow \pi N, \eta N, K\Lambda, K\Sigma$

- Differential cross section



[1] McCracken *et al.* 2010 (JLab), [2] Glander *et al.* 2004 (ELSA)

- Recoil polarization



FRI Heras *et al.* 2007 (CRAAL)

After this ...

- More double polarization observables in meson photoproduction to be published in the near future
- $\gamma N \rightarrow K\Sigma$
- Two meson photoproduction e.g. $\gamma p \rightarrow \pi^0 \eta p$ from ELSA



Challenges in Baryon Analysis

- Set common ground for different analyses – definite answers needed!
- Better $\pi N \rightarrow \pi N, \pi\pi N \eta N, KY$ data to improve analysis of photo- and electroproduction.
- Consistent data → Get rid of weighting data sets in fits (otherwise all statistical meaning lost).
- GWU/SAID: Provide correlations so other group can carry out statistically meaningful analyses.
- Answer the question: How sensitive are helicity couplings to the new set of XY data? Maybe: Eigenvectors ordered by eigenvalues of the inverse correlation matrix

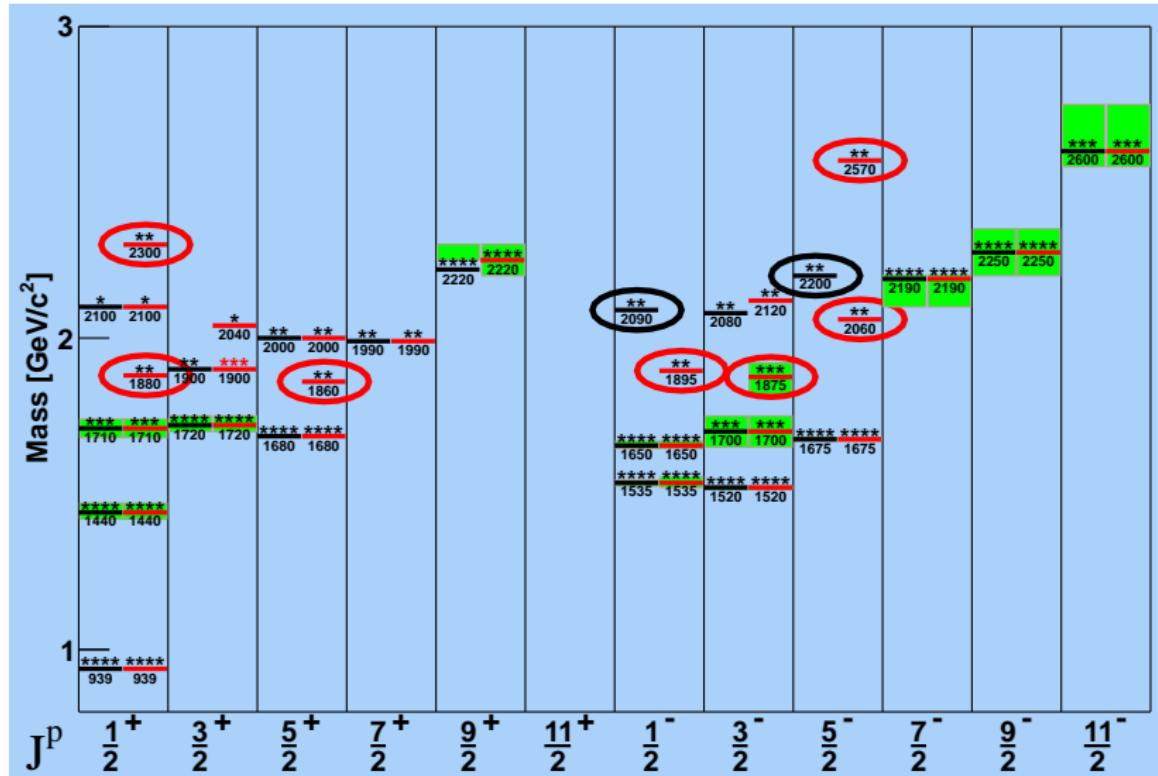
$$\frac{1}{2} \left(\frac{\partial^2 \chi_{XY}^2}{\partial A_i \partial A_j} \right) \quad (1)$$

A_i : i-th helicity coupling $|A|$ or complex phase θ ($A = |A| \exp(i\theta)$) at the pole.

- Approaching the precision frontier in baryon analysis:
Joint effort by GWU/INS SAID, Jülich, JPAC.

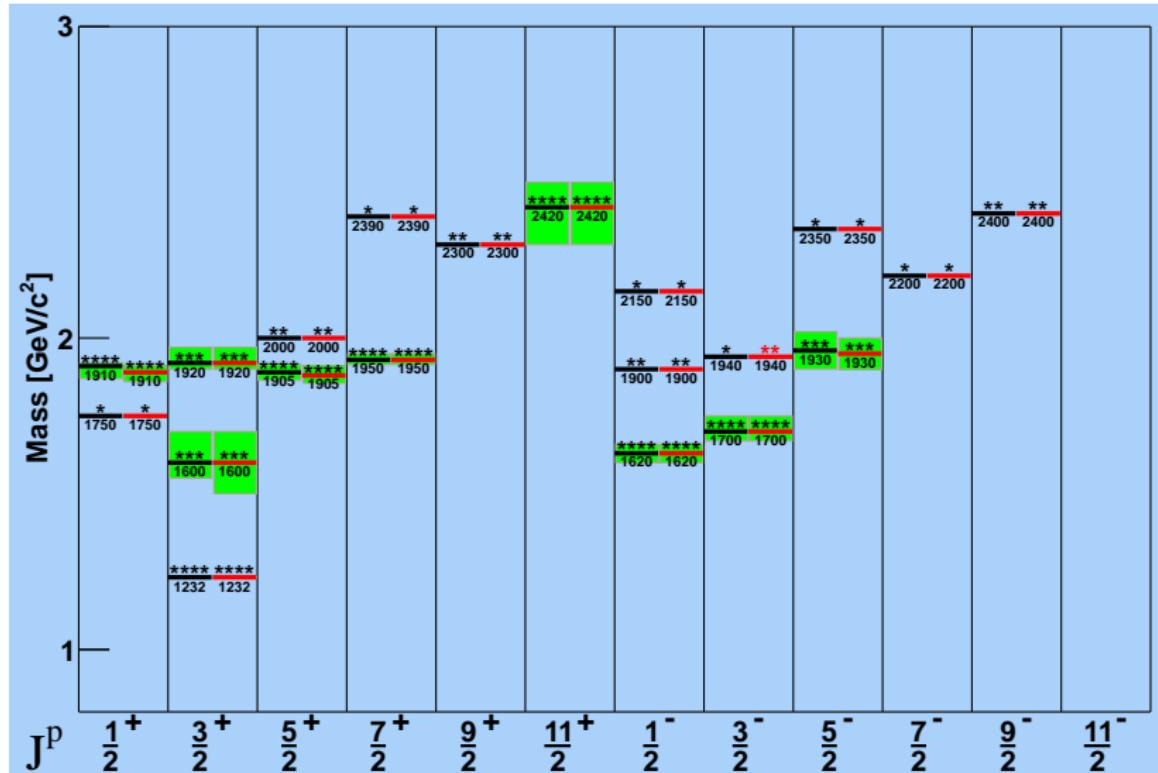
Baryon updates 2010 vs 2014

left: 2010, right: 2014. Courtesy of A. Wilson



Baryon updates 2010 vs 2014

left: 2010, right: 2014. Courtesy of A. Wilson

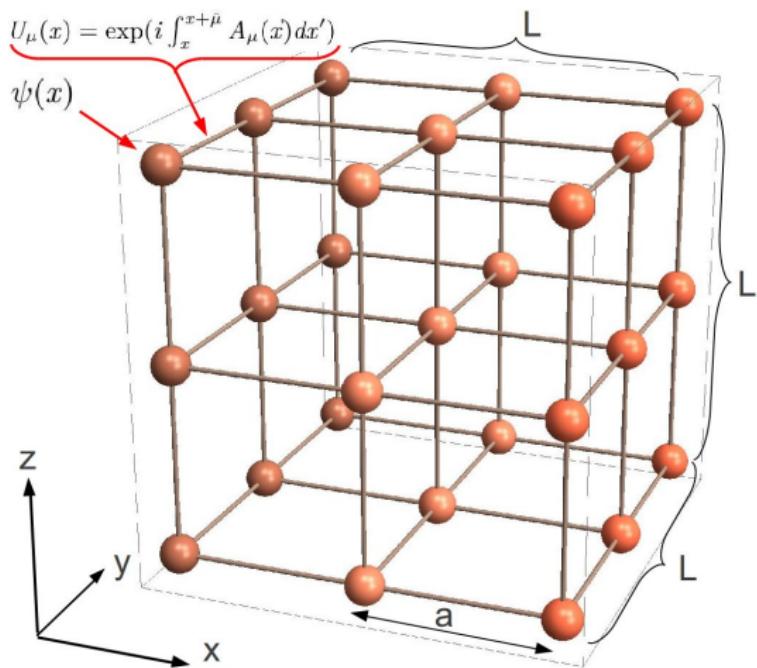


Connecting phenomenology to QCD

- How can QCD simulations be matched to the rich phenomenology of excited baryons?
 - Are there explicit experimental signatures of gluon dynamics at intermediate energies? Do exotics exist?
-

- Analysis of excited baryons on the lattice
- Analysis of excited mesons on the lattice

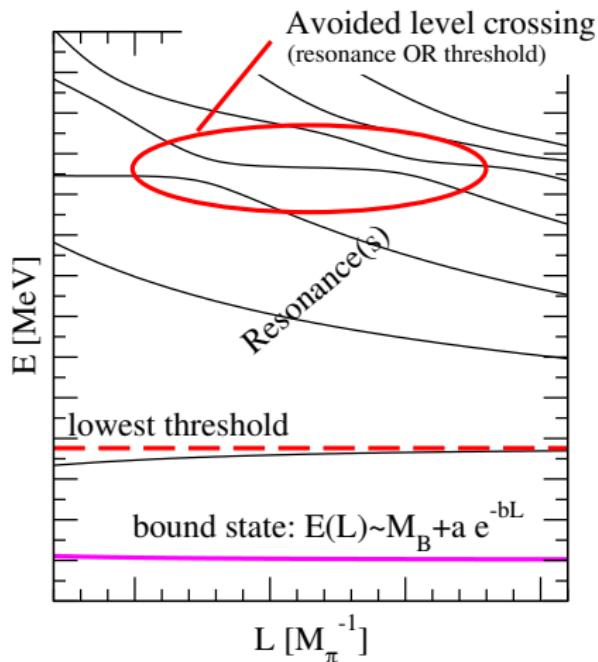
The cubic lattice



- Side length L ,
periodic boundary conditions
 $\Psi(\vec{x}) \stackrel{!}{=} \Psi(\vec{x} + \hat{\mathbf{e}}_i L)$
→ finite volume effects
→ Infinite volume $L \rightarrow \infty$
extrapolation
- Lattice spacing a
→ finite size effects
Modern lattice calculations:
 $a \simeq 0.07 \text{ fm} \rightarrow p \sim 2.8 \text{ GeV}$
→ (much) larger than typical hadronic scales;
not considered here.
- Unphysically large quark/hadron masses
→ (chiral) extrapolation required.

Resonances decaying on the lattice

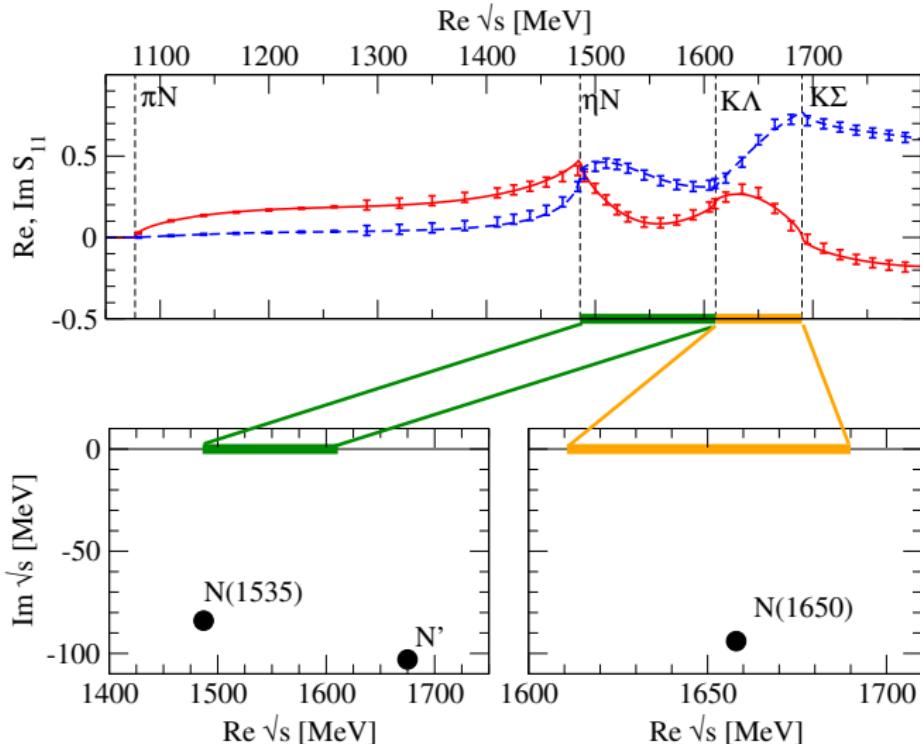
Eigenvalues in the finite volume



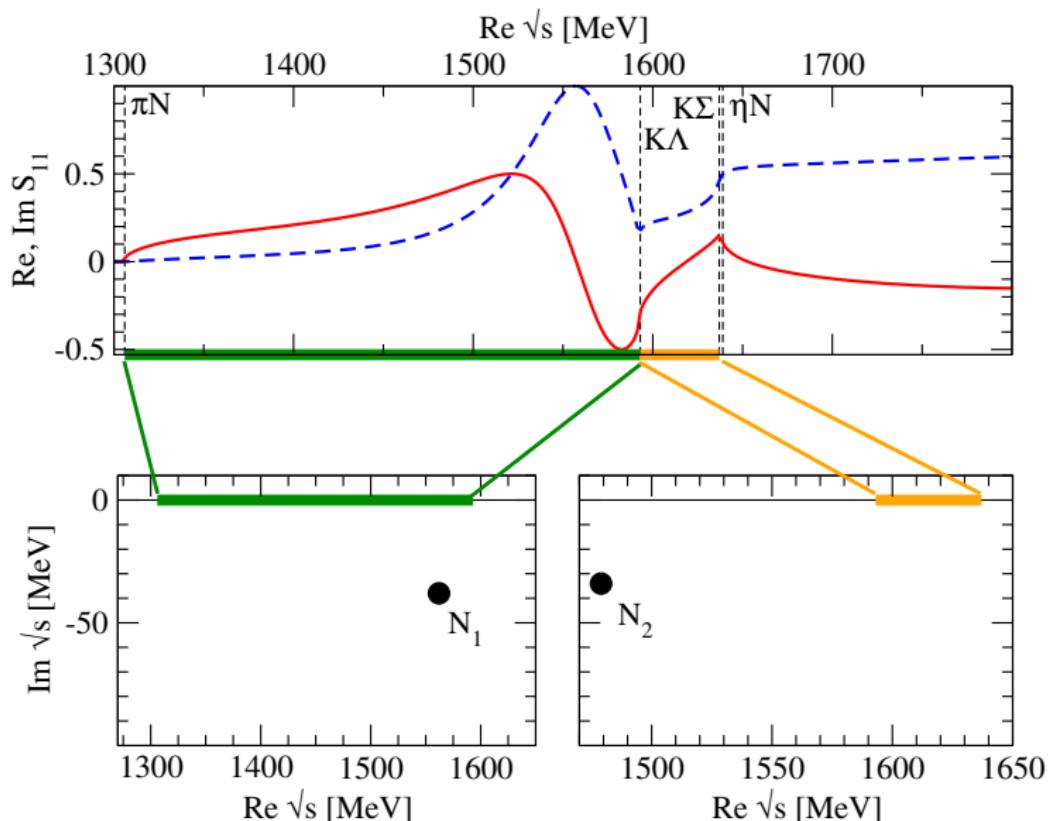
$J^P = 1/2^-$ in the finite volume

[M.D., Mai, Meißner, EPJA (2013)]

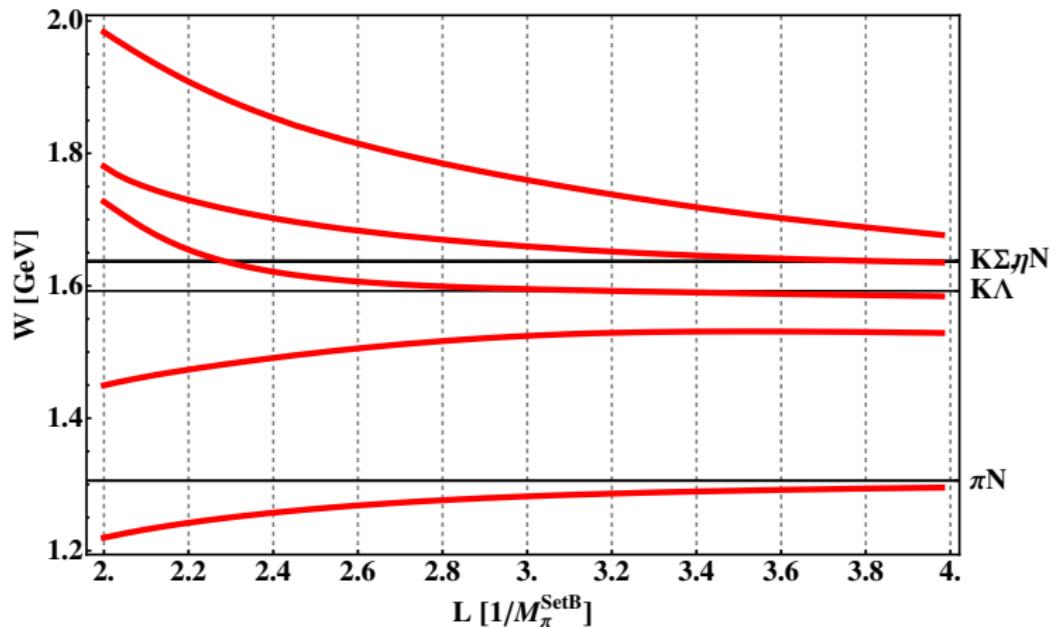
- Unitarized chiral interaction with NLO contact terms



Chiral extrapolation to a QCDSF lattice setup

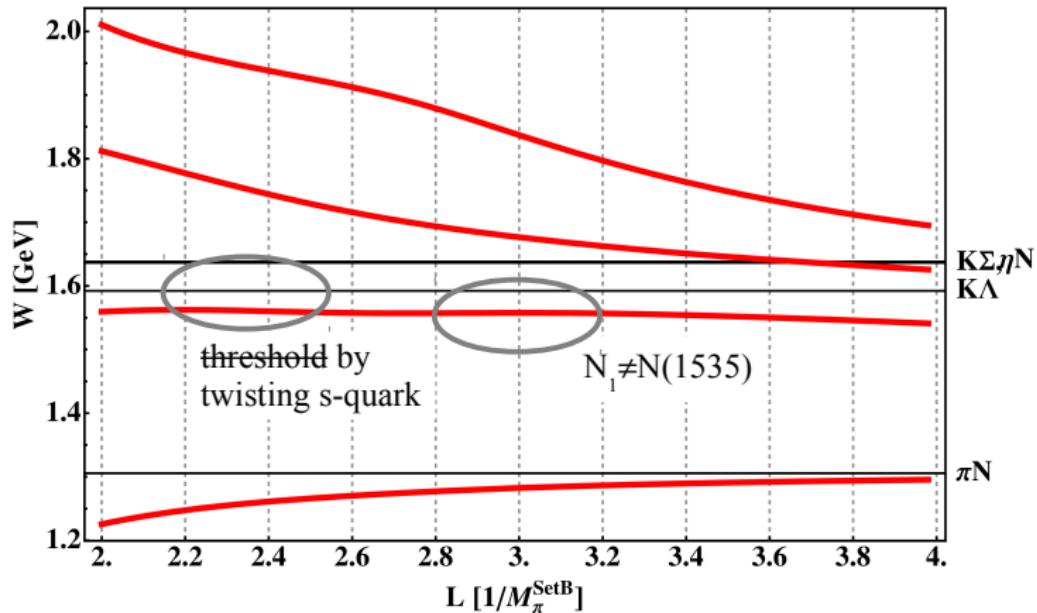


Prediction of the lattice spectrum



- No one-to-one mapping of levels to resonances → coupled channel analysis; hidden poles appear.

Prediction of the lattice spectrum

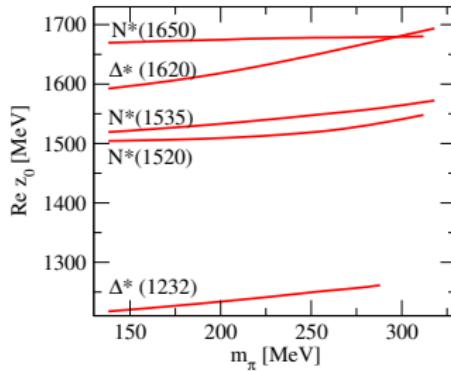
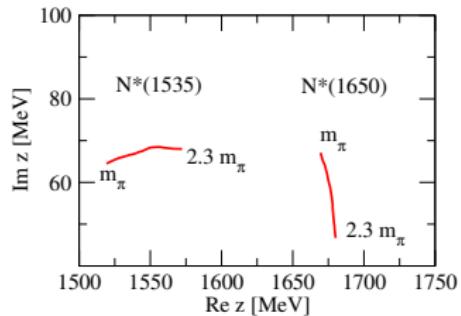
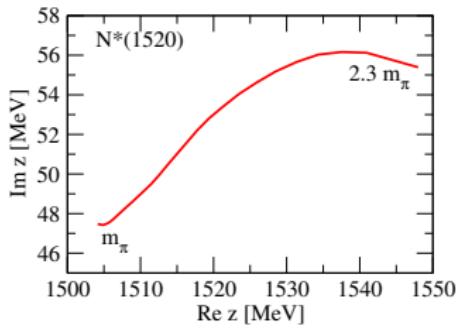


- No one-to-one mapping of levels to resonances → coupled channel analysis; hidden poles appear. **Aperiodic Boundary Conditions to reveal eigenlevel more clearly.**

Chiral extrapolation from dynamical coupled channels approaches

based on Jülich solution NPA 829, 170 (2009)

- Larger theoretical uncertainties.
- Quark mass dependence of effective $2\pi N$ channels is intricate.
- Finite-volume effects of 3-body channels unexplored.
- Accessing the Roper puzzle.



N π (1/2 $^{-}$) channel

$m_\pi = 266$ MeV; distillation method; variational analysis using a basis of N (3 quarks) and N π (5 quarks) interpolators;

$$(N_{\pm}^{(i)})_{\mu}(\vec{p} = 0) = \sum_{\vec{x}} \epsilon_{abc} \left(P_{\pm} \Gamma_1^{(i)} u_a(\vec{x}) \right)_{\mu} \left(u_b^T(\vec{x}) \Gamma_2^{(i)} d_c(\vec{x}) \right)$$

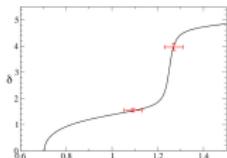

$$\pi^+(\vec{p} = 0) = \sum_{\vec{x}} \bar{d}_a(\vec{x}) \gamma_5 u_a(\vec{x}),$$


$$\pi^0(\vec{p} = 0) = \sum_{\vec{x}} \frac{1}{\sqrt{2}} (\bar{u}_a(\vec{x}) \gamma_5 u_a(\vec{x}) - \bar{d}_a(\vec{x}) \gamma_5 d_a(\vec{x}))$$

$$O_{N\pi}(I = \frac{1}{2}, I_3 = \frac{1}{2}) = p\pi^0 + \sqrt{2} n\pi^+$$

$$N\pi(\vec{p} = 0) = \gamma_5 N_+(\vec{p} = 0)\pi(\vec{p} = 0)$$

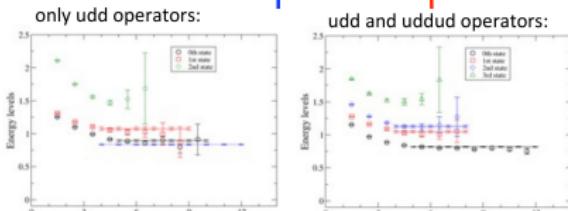
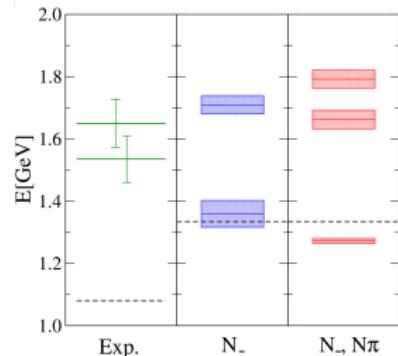

Lüscher relation
→ phase shift:



Assuming 2 elastic resonances with identical coupling we get $m_1 = 1.678$ GeV

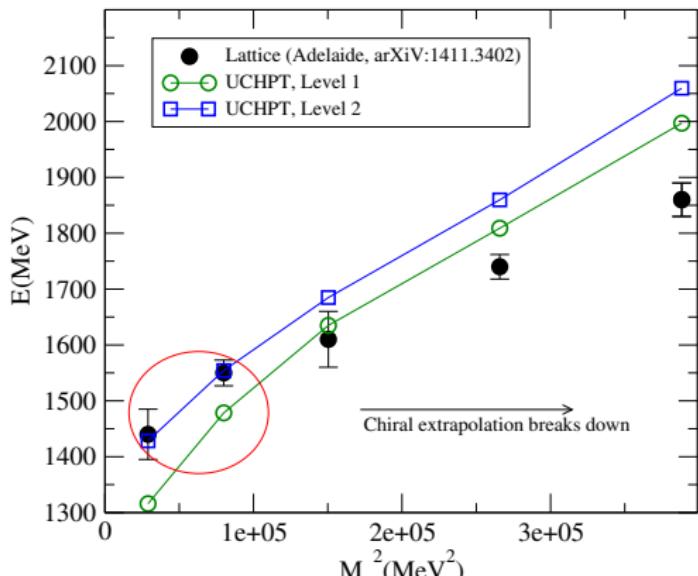
$m_2 = 1.873$ GeV

C.B. Lang and V. Verduci,
Phys. Rev. D 87, 054502 (2013)



The $\Lambda(1405)$: Predictions from Unitarized CHPT

Raquel Molina, M.D.



- Coupled channels $\pi\Sigma$, $\bar{K}N$, $\eta\Lambda$, $K\Xi$
- Unitarized LO χ potential V in $T = V + VGT$
- No freedom at this order → full prediction.
- UCHPT has two poles for the $\Lambda(1405)$.
- → new data not in conflict with two-pole structure of a molecular $\Lambda(1405)$ (but not yet a proof thereof).
- → next step: Statistical NLO analysis.

Summary and Outlook

Extraction of the N^* and Δ resonance spectrum

- DCC analysis of $\pi N \rightarrow \pi N, \eta N, K\Lambda$ and $K\Sigma$

Jülich model: lagrangian based, unitarity & analyticity respected

→ analysis of over 6000 data points (PWA, $d\sigma/d\Omega, P, \beta$)

→ extraction of resonance parameters (**poles & residues**)

- Pion photoproduction in a semi-phenomenological approach

hadronic final state interaction: Jülich DCC analysis

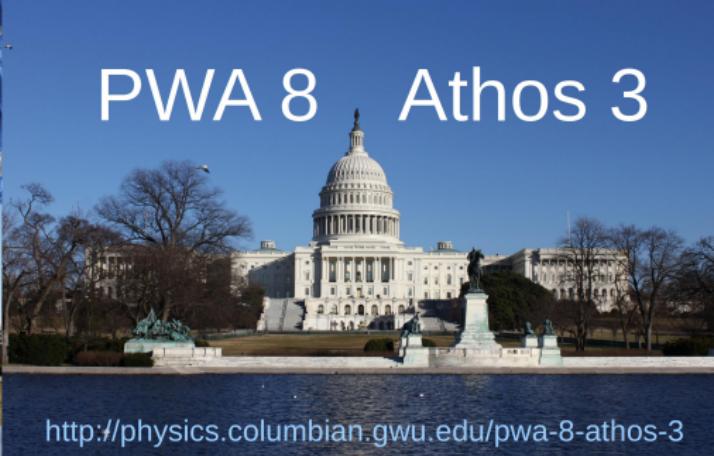
→ analysis of more than 23 000 data points for **single** and **double polarization** observables

→ extraction of **photocouplings at the pole**

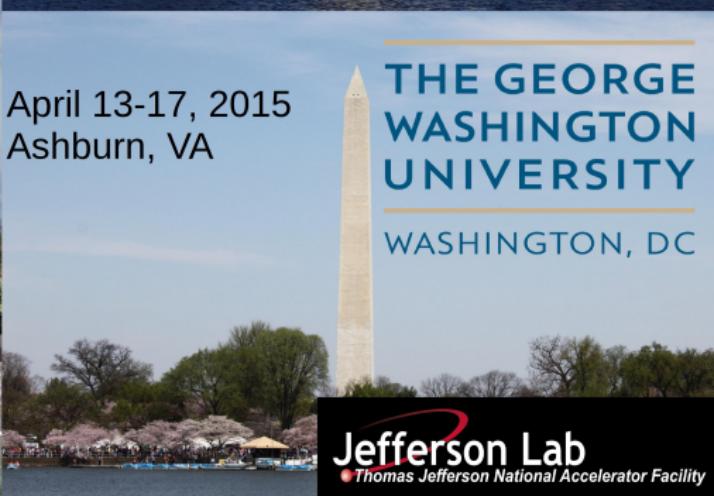
- Next: η (world data fitted) and Kaon photoproduction (in progress)

- Next: Analyze data at (un)physical quark masses simultaneously.

PWA 8 Athos 3



<http://physics.columbian.gwu.edu/pwa-8-athos-3>



April 13-17, 2015
Ashburn, VA

THE GEORGE
WASHINGTON
UNIVERSITY
WASHINGTON, DC



Jefferson Lab
Thomas Jefferson National Accelerator Facility

PWA 8 / ATHOS 3

The International Workshop on Partial Wave Analysis for Hadron Spectroscopy

The International Workshop on Partial Wave Analysis for Hadron Spectroscopy (PWA 8/ATHOS 3) will take place in Ashburn, Virginia, from April 13-17, 2015.

The goal of the workshop is to bring together experts from the experimental and theoretical community to discuss the current and future issues in the field.

The format of the Workshop is based on plenary sessions including overview talks, a number of topical presentations and several discussion sessions with the aim of summarizing the state-of-the-art in hadron spectroscopy, highlighting the open questions and discussing the development of new tools for next generation experiments.

Local Organizing Committee:

William J. Briscoe (GWU)

Michael Doring (GWU)

Helmut Haberzettl (GWU)

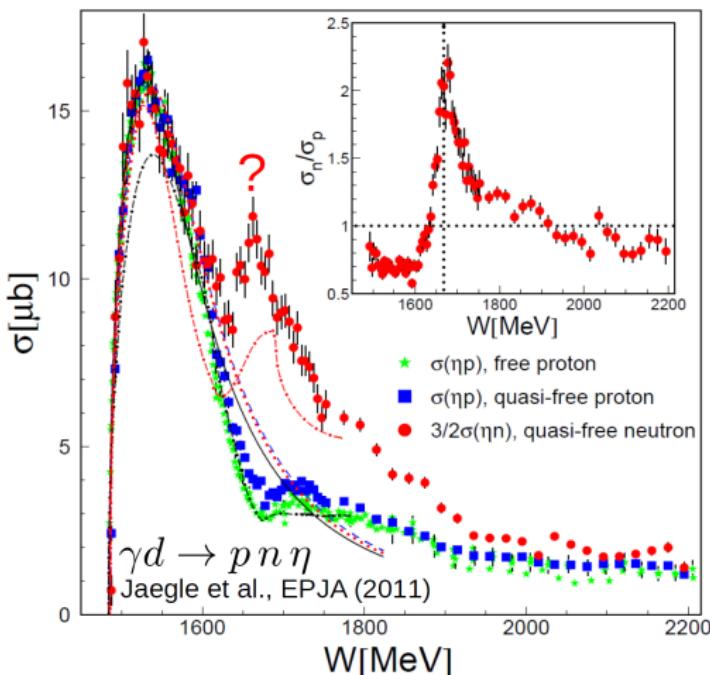
Michael Pennington (JLab)

Adam Szczepaniak (Indiana U./JLab)

Contact: Tel.: +1 (202) 994 8578, +1(703) 554 9495, pwaathos@gwu.edu

[Download the First Circular](#)

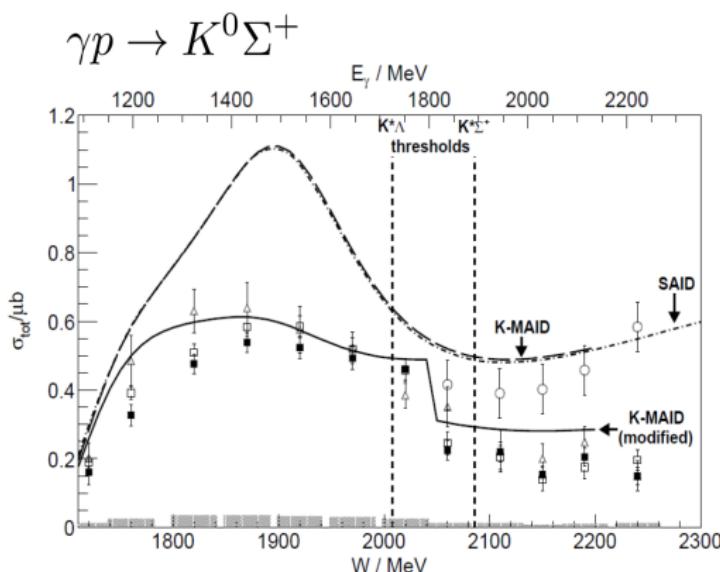
Phenomenological challenges: bump in “ $\gamma n \rightarrow \eta n$ ” (MAMI)



- “ $N(1685)$ ”: large couplings to $\gamma n, \eta n$
- Pentaquark? [Prediction Polyakov et al.]
- Unusual combination of neutron helicity couplings? [BnGa]
- Coupled channel chiral dynamics from $K\Lambda, K\Sigma$? [M.D. et al.]

- MAMI: Especially designed for neutral final states.
- Excellent energy resolution & angular coverage
- $\pi^0 p$ final state needed for isospin decomposition in PWA; complementary to CLAS ($\pi^+ n$).
- Strong **involvement** of US physicists (GWU, KSU,...)

Phenomenological challenges: $K\Sigma$ photoproduction (ELSA)



CBELSA/TAPS [PLB 2012]

- Sudden drop at $W \sim 2.05 \text{ GeV}$
- Dynamic effect
[Ramos/Oset, PLB 2013]?
- CLAS, MAMI, ELSA:
Strong complementary experimental program
- Joint data essential for partial wave analysis
- Still surprises in meson photoproduction.

Analysis of pion-induced reactions

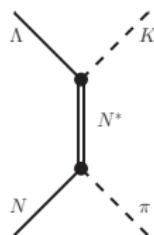
Eur.Phys.J. A49 (2013) 44, Nucl.Phys. A851 (2011) 58-98

- calculate observables from T -matrix
- fit free parameters of T to data or partial wave amplitudes

$$\sigma = \frac{1}{2} \frac{4\pi}{p^2} \sum_{JLS, L'S'} |\tau_{LS}^{JL'S'}|^2$$

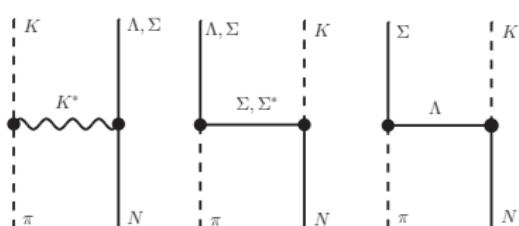
with $\tau_{fi} = -\pi \sqrt{\rho_f \rho_i} T_{fi}$
 ρ : phase factor

s-channel: **resonances** (T^P)



$$m_{bare} + f_{\pi NN^*}$$

t- and u-channel exchange: "background" (T^{NP})



cut off Λ in form factors $\left(\frac{\Lambda^2 - m_{ex}^2}{\Lambda^2 + q^2} \right)^n$
(couplings fixed from SU(3))

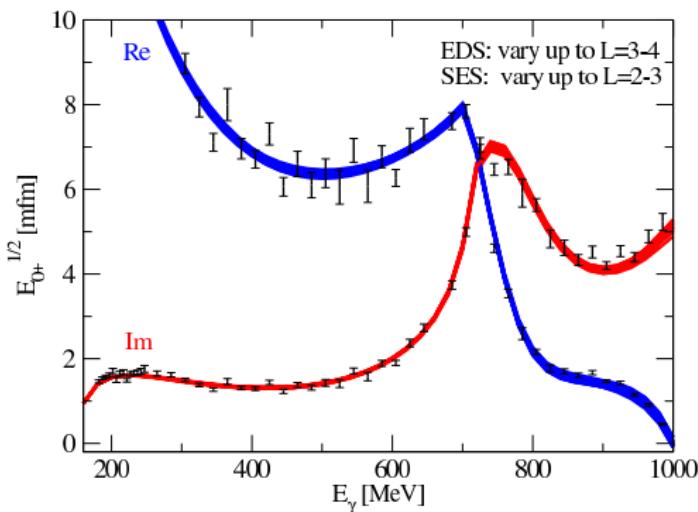
⇒ search for poles in the complex energy plane of T

Putting different analyses on common ground

... and have eventually convergence of results among different groups (?)

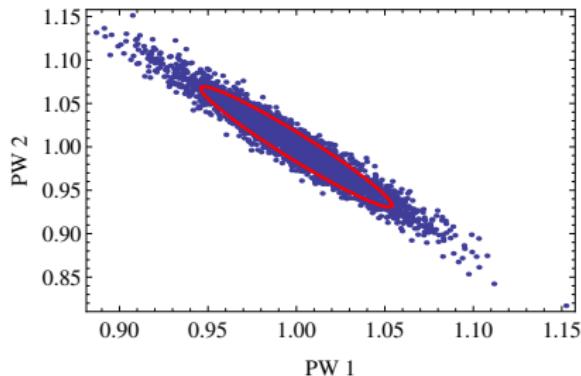
- BnGa, Jülich, MAID, Zagreb... fit to SAID elastic πN partial waves or use it as FSI.
- Single-energy solutions of SAID: narrow structures & consistency, but little statistical meaning.
- No control of the statistical impact of elastic πN data on multi-reaction fits, performed by other groups.
- Q: is it possible to provide a simple to use interface, such that other groups can fit to SAID elastic πN PWs and get the same χ^2 as if they had fitted to the elastic πN data?

Work in progress: Monte-Carlo error propagation



- Bands: Monte-Carlo error propagation using bootstrap (energy-dependent fit).
- Statistically meaningful
- but: correlations between different partial waves/multipoles are also there!
- How to provide? Solution: Instead of PWs, we plan to provide correlation matrices for the SES.

Work in progress: Correlated χ^2 fits



Fit “observable” $O(E)$ with

$$O(E) = (\text{PW 1})E + (\text{PW 2}) \frac{E^2}{100}$$

Data generated around

$$O(E) = E + \frac{E^2}{100}$$

- Stochastic estimate of correlation matrix.
- Correlated χ^2 : $\chi^2(x) = (x - \bar{x})^T C^{-1}(x - \bar{x})$, $x = (S_{11}, S_{31}, \dots)$
- but does this provide the same χ^2 as a fit to the original data?

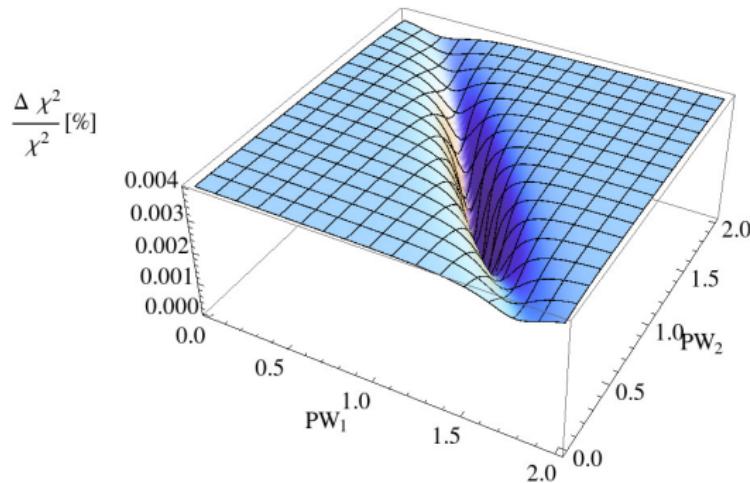
Work in progress: Correlated χ^2 fits

- ... almost; the function

$$\chi^2 = (x - \bar{x})^T C^{-1} (x - \bar{x}) + \chi_{\text{best}}^2 + \text{corrections}$$

does.

- Difference between correlated χ^2 and actual χ^2 of the data themselves (toy model):



Data base and fit

Data base:

- elastic πN PWA SAID 2006 [Arndt et al., PRC 74 (2006)]
 - $\pi N \rightarrow \eta N, KY$ observables ($d\sigma/d\Omega, P, \beta$)
- $\left. \right\} \sim 6000$ data points

Fit parameters:

- T^P : 128 free parameters
 - $11 N^*$ resonances $\times (1 m_{bare} + \text{couplings to } \pi N, \rho N, \eta N, \pi\Delta, K\Lambda, K\Sigma))$
 - $+ 10 \Delta$ resonances $\times (1 m_{bare} + \text{couplings to } \pi N, \rho N, \pi\Delta, K\Sigma)$
 - T^{NP} : 68 free parameters
- $\left. \right\} 196$ free parameters

⇒ Simultaneous fit of T^P and T^{NP} using MINUIT on the JUROPA supercomputer (FZJ)

Sensitivity of results to starting conditions?

⇒ Perform 2 fits, start from different scenarios in parameter set

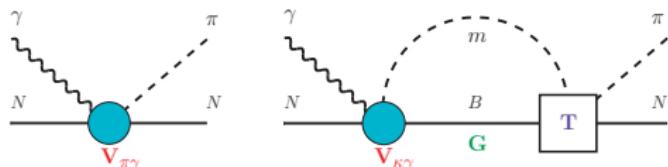
- Fit A: start from parameter set of a preceding Jülich model [Gasparyan et al. PRC68 , (2003)]
- Fit B: start from a new scenario

Photoproduction in a semi-phenomenological approach

Multipole amplitude

$$M_{\mu\gamma}^{IJ} = V_{\mu\gamma}^{IJ} + \sum_{\kappa} T_{\mu\kappa}^{IJ} G_{\kappa} V_{\kappa\gamma}^{IJ}$$

(partial wave basis)



$$m = \pi, \eta, B = N, \Delta$$

$T_{\mu\kappa}$: Jülich hadronic T -matrix

→ Watson's theorem fulfilled by construction

→ analyticity of T : extraction of resonance parameters

Phenomenological potential:

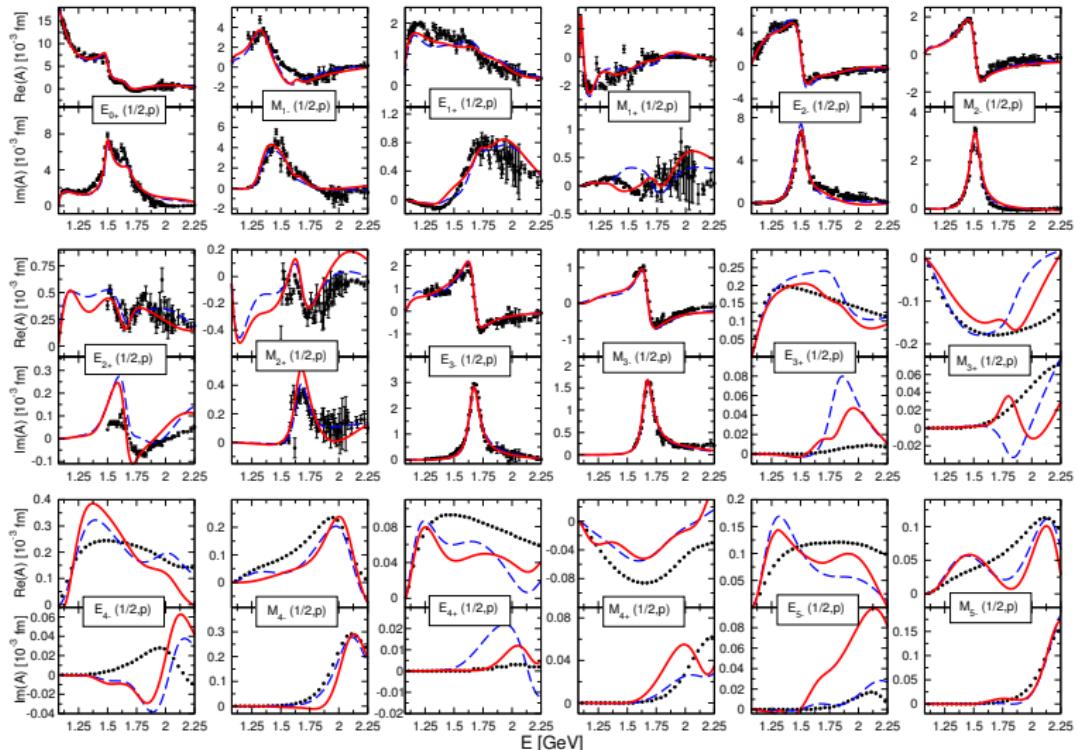
$$\begin{aligned} \mathbf{V}_{\mu\gamma}(E, q) &= \text{---} + \text{---} = \frac{\tilde{\gamma}_{\mu;i}^a(q)}{m_N} P_{\mu}^{\text{NP}}(E) + \sum_i \frac{\gamma_{\mu;i}^a(q) P_i^{\text{P}}(E)}{E - m_i^b} \\ &\quad \text{---} \quad \text{---} \end{aligned}$$

The diagram shows the decomposition of the total potential $\mathbf{V}_{\mu\gamma}$ into a contact term (represented by a wavy line and a dot) and a pole term (represented by a wavy line and a horizontal line with a resonance exchange). The contact term is proportional to P_{μ}^{NP} . The pole term involves a nucleon (N) and a baryon (B) exchange, with a resonance (N^*, Δ^*) at the vertex. The hadronic vertices are labeled $\tilde{\gamma}_{\mu;i}^a(q)$ and $\gamma_{\mu;i}^a(q)$.

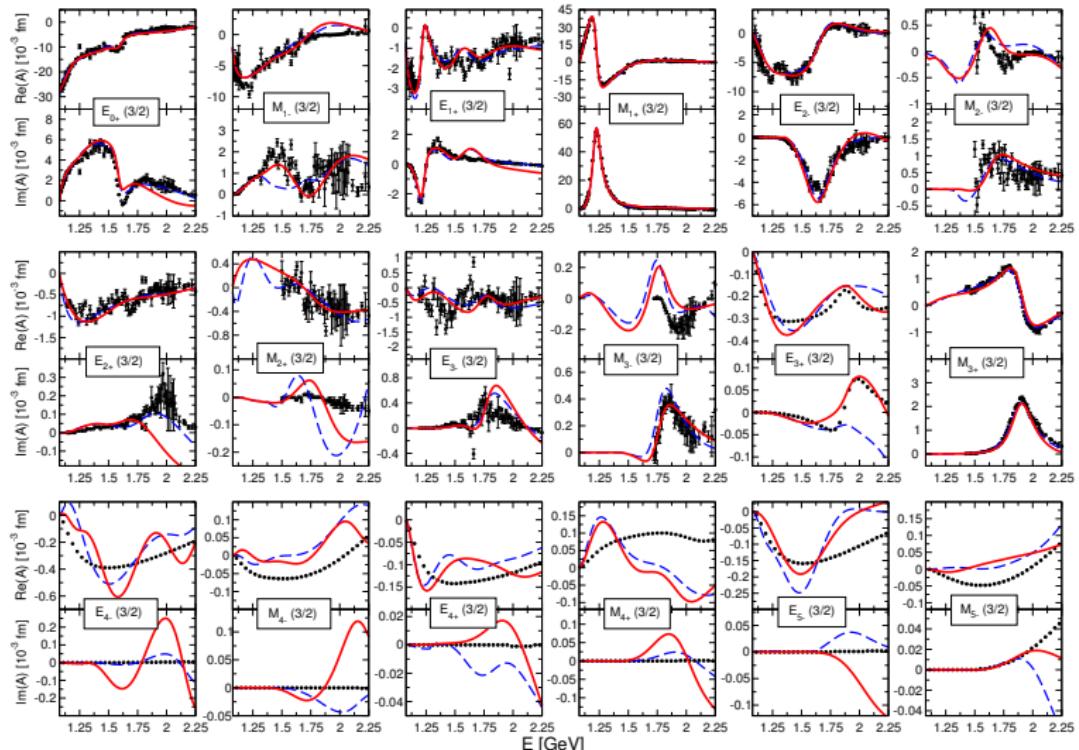
$\tilde{\gamma}_{\mu;i}^a, \gamma_{\mu;i}^a$: hadronic vertices → correct threshold behaviour
 i : resonance number per multipole; μ : channels $\pi N, \eta N, \pi \Delta$

Method inspired by SAID
but different

Multipoles: Comparison with GWU/SAID CM12



Multipoles: Comparison with GWU/SAID CM12



Details of the formalism

Polynomials:

$$P_i^P(E) = \sum_{j=1}^n g_{i,j}^P \left(\frac{E - E_0}{m_N} \right)^j e^{-g_{i,n+1}^P(E-E_0)}$$

$$P_\mu^{NP}(E) = \sum_{j=0}^n g_{\mu,j}^{NP} \left(\frac{E - E_0}{m_N} \right)^j e^{-g_{\mu,n+1}^{NP}(E-E_0)}$$

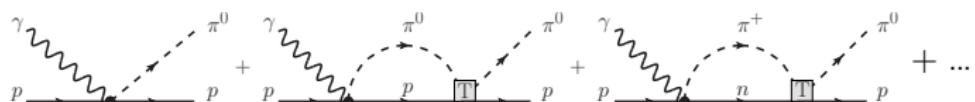
- $E_0 = 1077$ MeV
- $g_{i,j}^P, g_{\mu,j}^{NP}$: fit parameter
- $e^{-g(E-E_0)}$: appropriate high energy behavior
- $n = 3$

◀ back

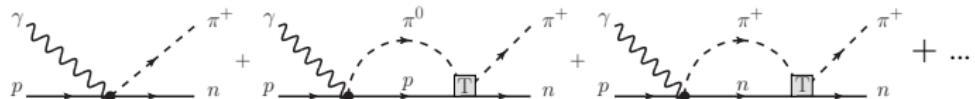
Isospin breaking in the πN channel

- Hadronic reactions: data well above threshold \rightarrow **isospin averaged masses**
- Pion-photoproduction: data near threshold \rightarrow include **isospin breaking**
 \hookrightarrow use **exact π^0 (π^+) and p (n) mass**

$E_{cm} < 1140$ MeV:

$$M_{\pi^0 p}^\gamma = \text{Diagram sequence for } \pi^0 p \text{ production}$$


The diagram shows a sequence of interactions between a photon (γ) and a proton (p). The first interaction produces a π^0 meson and a proton. Subsequent interactions involve the π^0 meson interacting with the proton, followed by another photon-proton interaction producing a π^0 meson and a proton. This sequence continues, with each interaction involving a photon, a π^0 meson, and a proton.

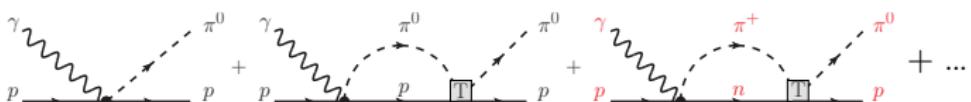
$$M_{\pi^+ n}^\gamma = \text{Diagram sequence for } \pi^+ n \text{ production}$$


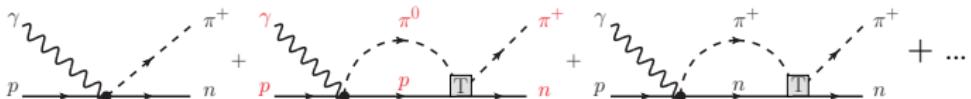
The diagram shows a sequence of interactions between a photon (γ) and a neutron (n). The first interaction produces a π^+ meson and a neutron. Subsequent interactions involve the π^+ meson interacting with the neutron, followed by another photon-neutron interaction producing a π^+ meson and a neutron. This sequence continues, with each interaction involving a photon, a π^+ meson, and a neutron.

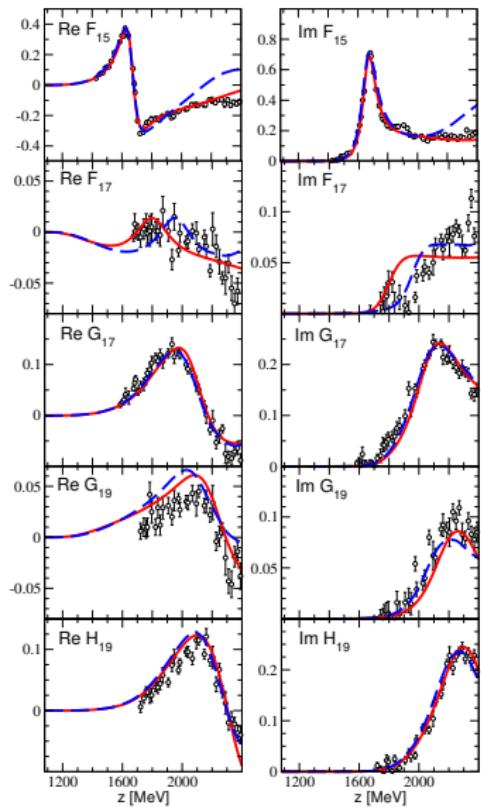
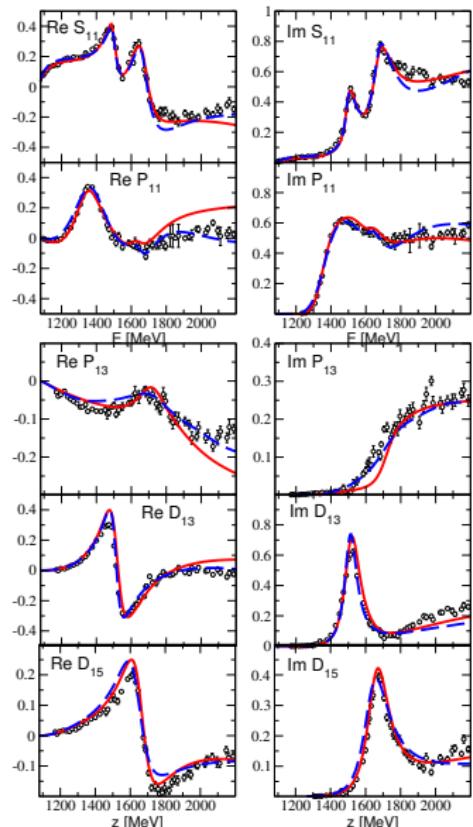
Isospin breaking in the πN channel

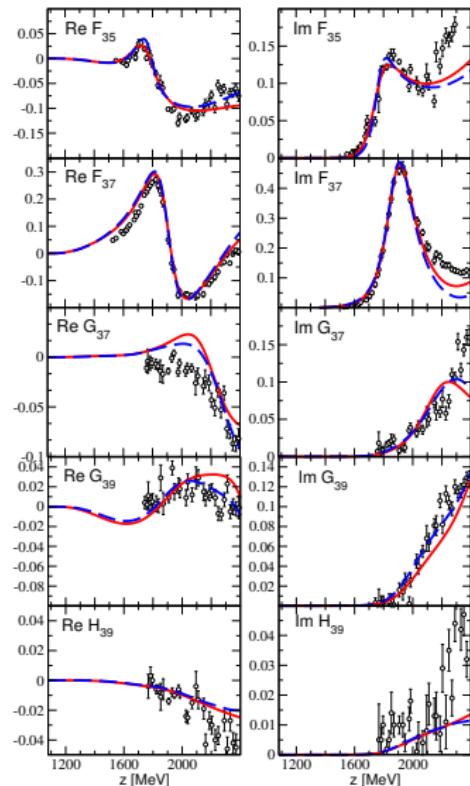
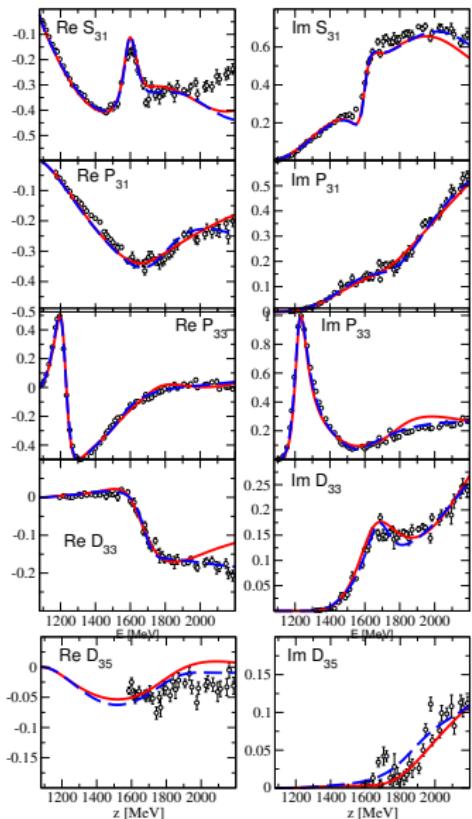
- Hadronic reactions: data well above threshold → **isospin averaged** masses
- Pion-photoproduction: data near threshold → include **isospin breaking**
↪ use exact π^0 (π^+) and p (n) mass

$E_{cm} < 1140$ MeV:

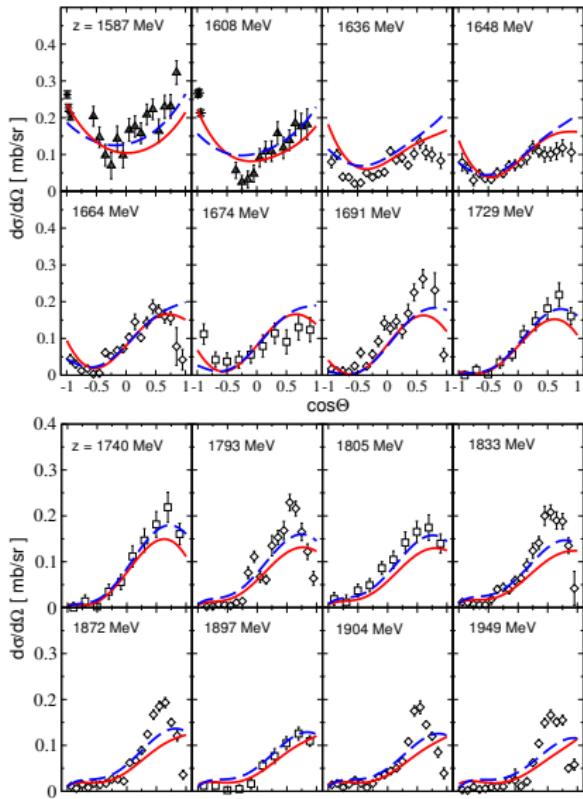
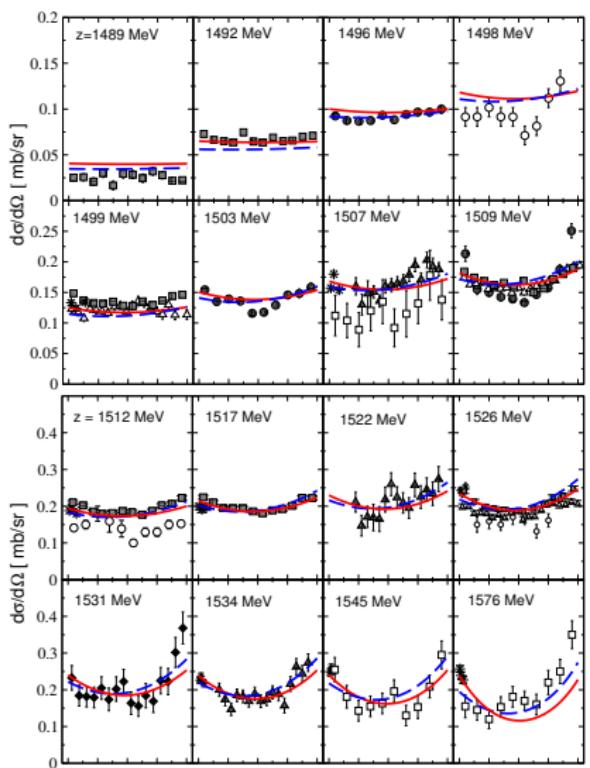
$$M_{\pi^0 p}^\gamma = \text{Diagram sequence for } \pi^0 p \text{ channel}$$


$$M_{\pi^+ n}^\gamma = \text{Diagram sequence for } \pi^+ n \text{ channel}$$


$\pi N \rightarrow \pi N$: Partial wave amplitudes $|l=1/2\rangle$


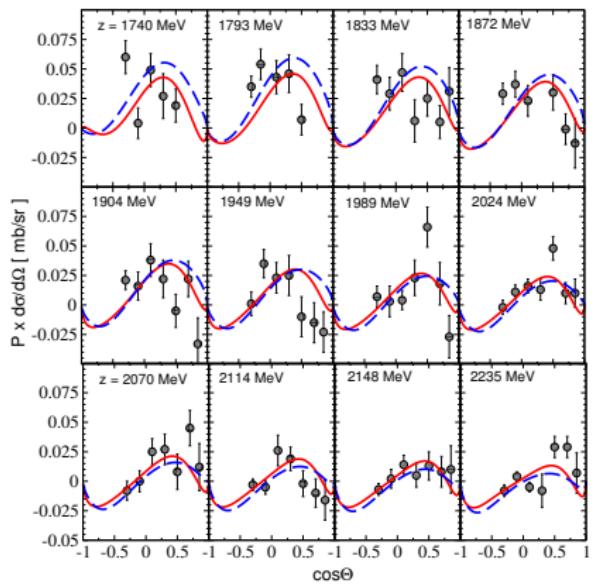
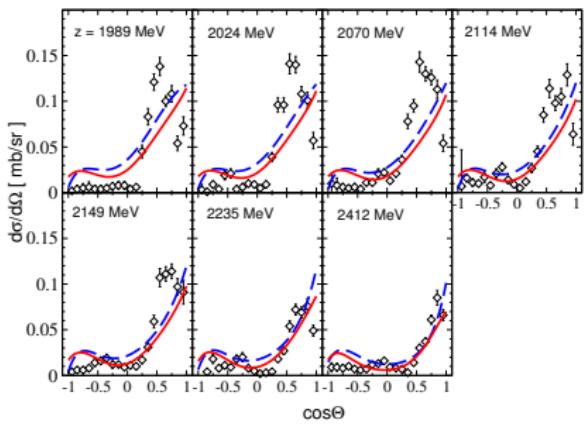
$\pi N \rightarrow \pi N$: Partial wave amplitudes $|l=3/2$


Full results ηN : $d\sigma/d\Omega$

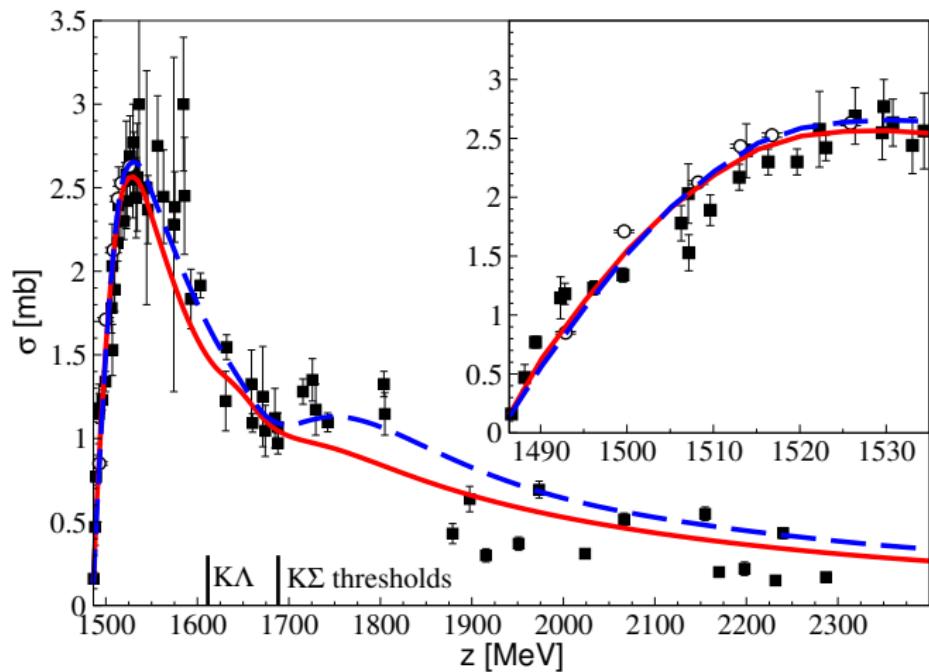


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Full results ηN : $d\sigma/d\Omega$ & Polarization

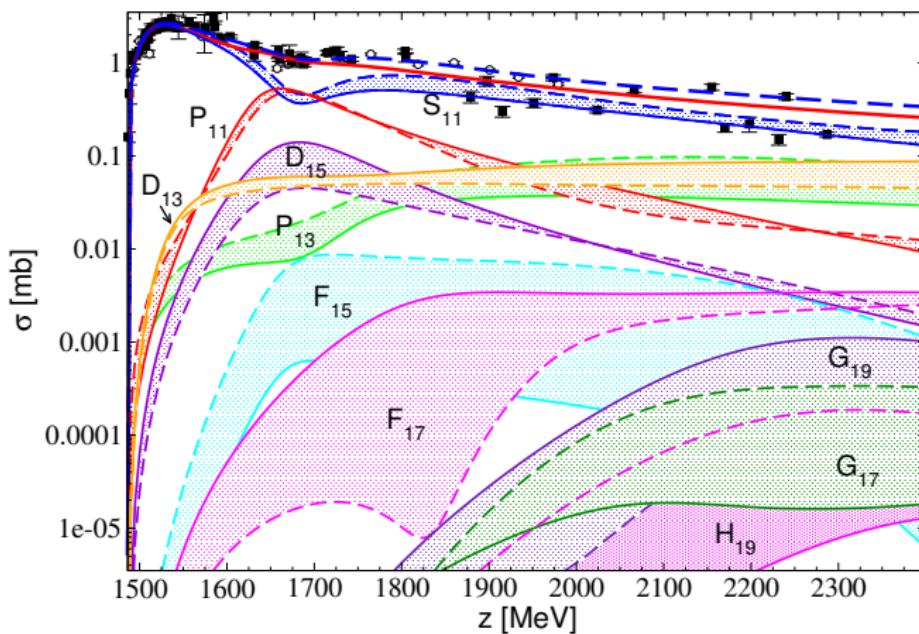


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$\pi^- p \rightarrow \eta N$: Total cross section

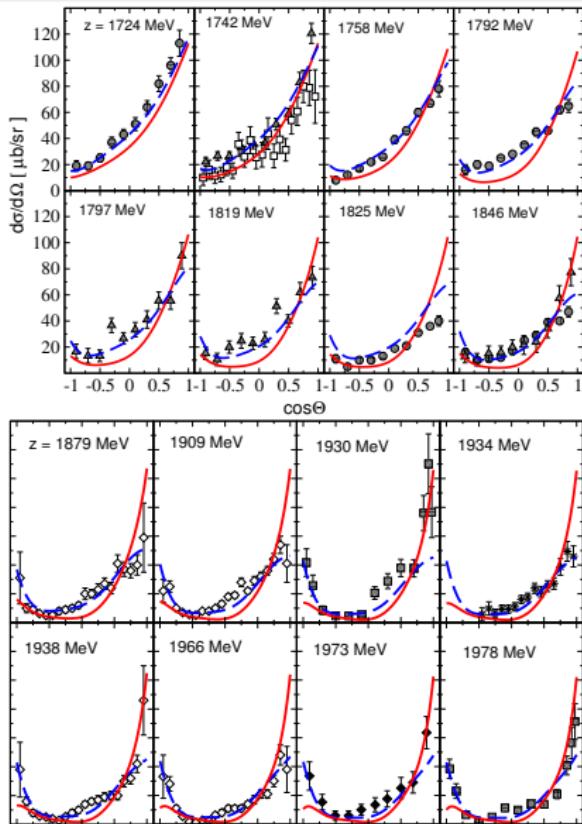
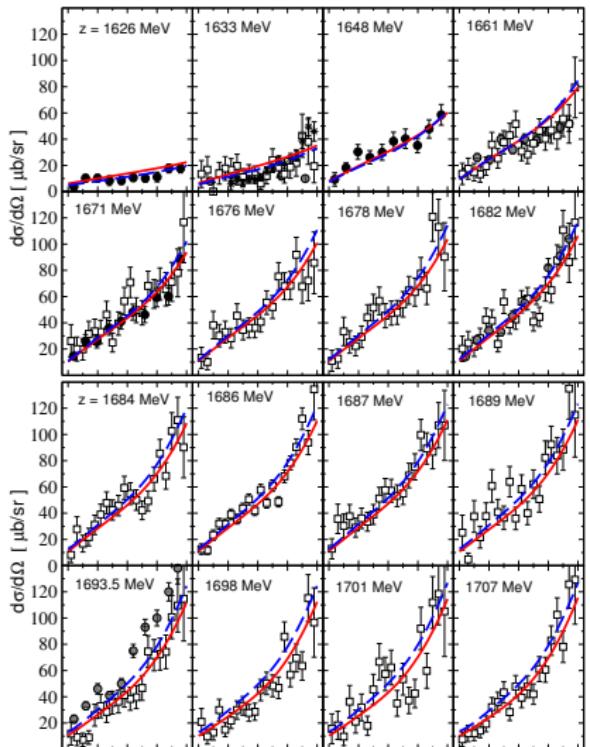
Partial wave content

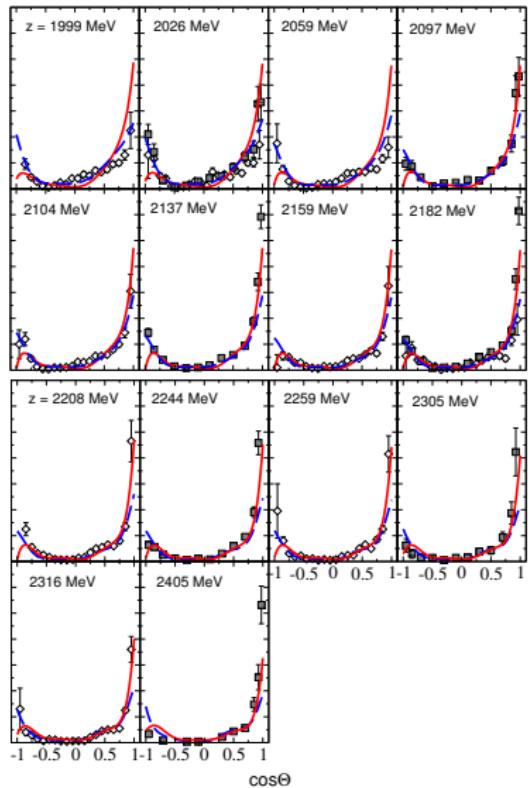
ηN : Partial wave content



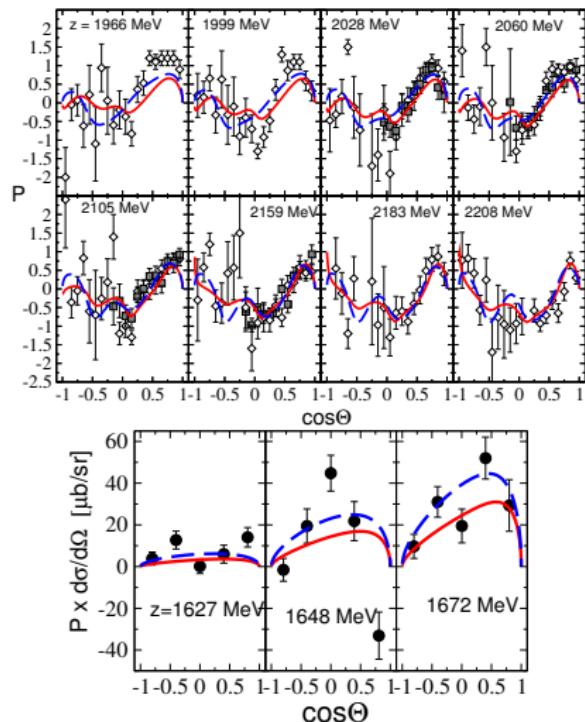
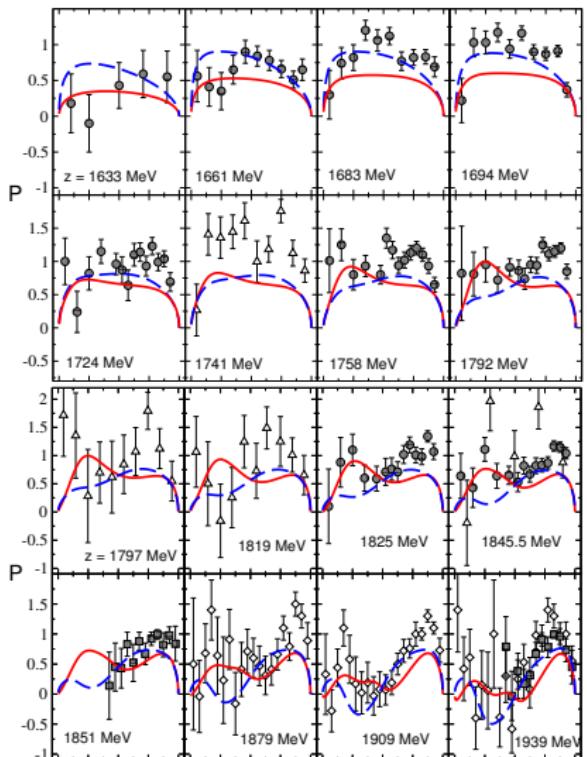
◀ back

Full results $K\Lambda$: $d\sigma/d\Omega$

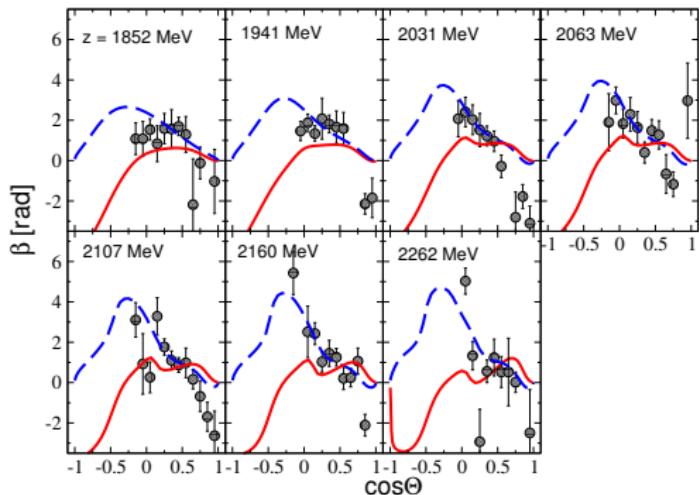


Full results $K\Lambda$: $d\sigma/d\Omega$ [◀ back](#)

Full results $K\Lambda$: Polarization



[◀ back](#)

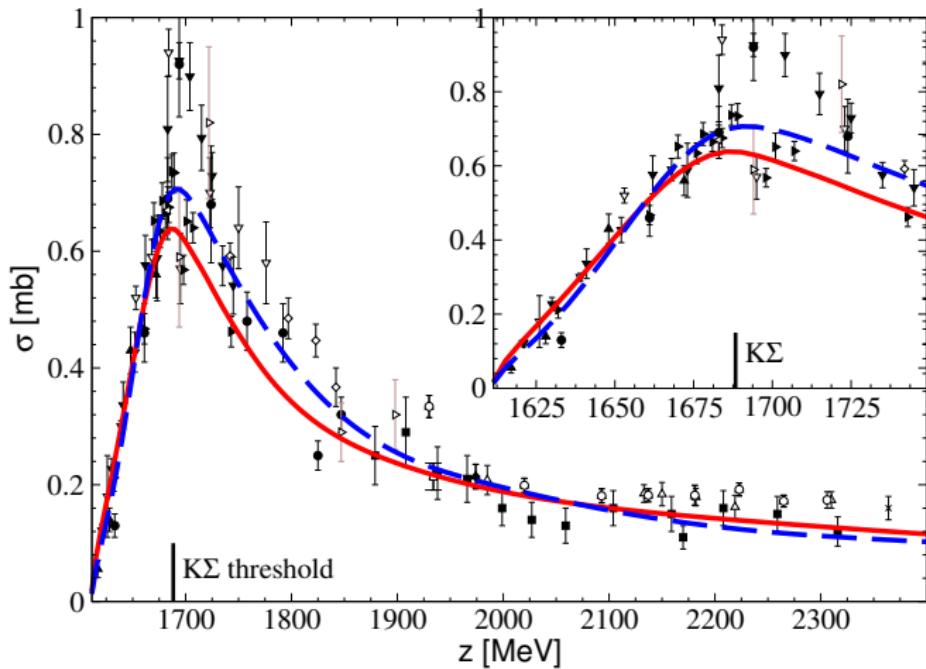
Full results $K\Lambda$: Spinrotation parameter β 

$$\beta = \arctan \left(\frac{2 \operatorname{Im}(h_{fi}^* g_{fi})}{|g_{fi}|^2 - |h_{fi}|^2} \right)$$

[◀ back](#)

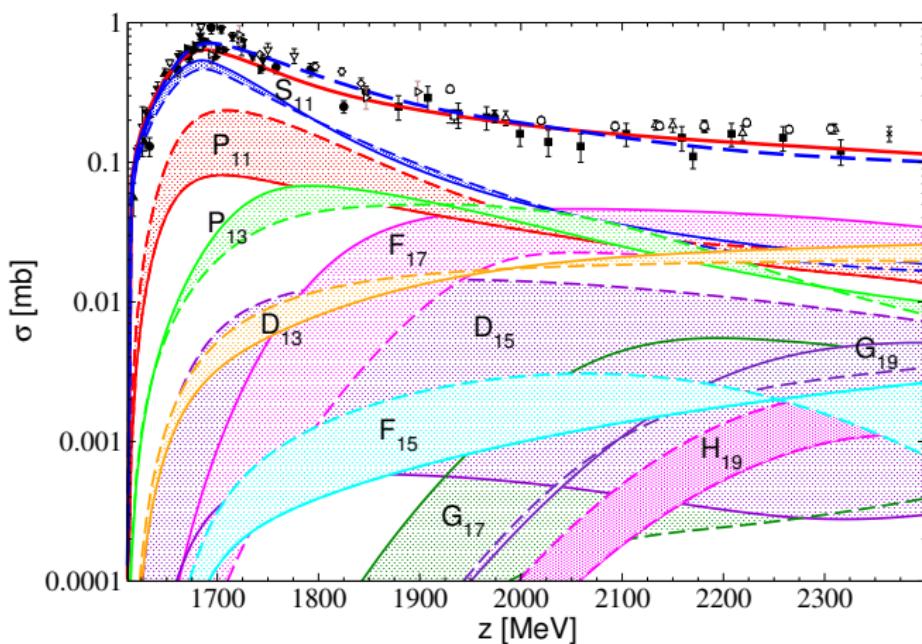
$\pi^- p \rightarrow K^0 \Lambda$: Total cross section

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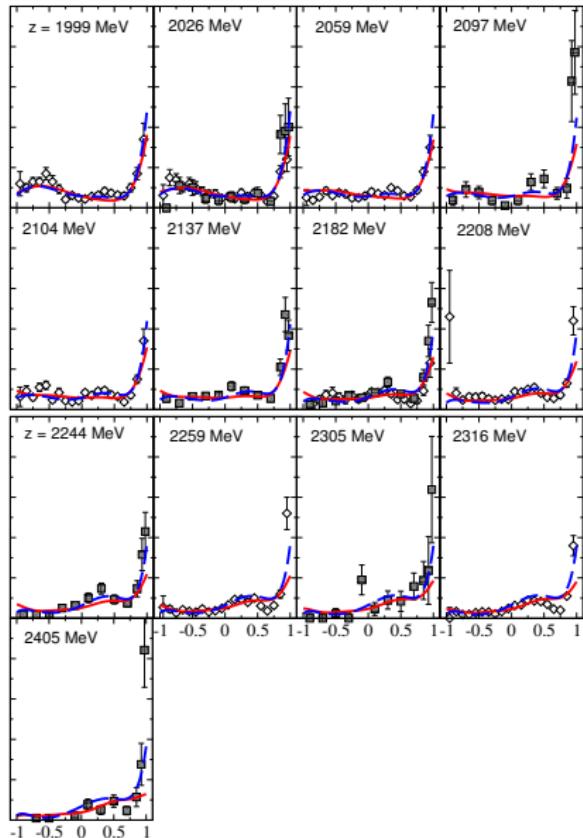
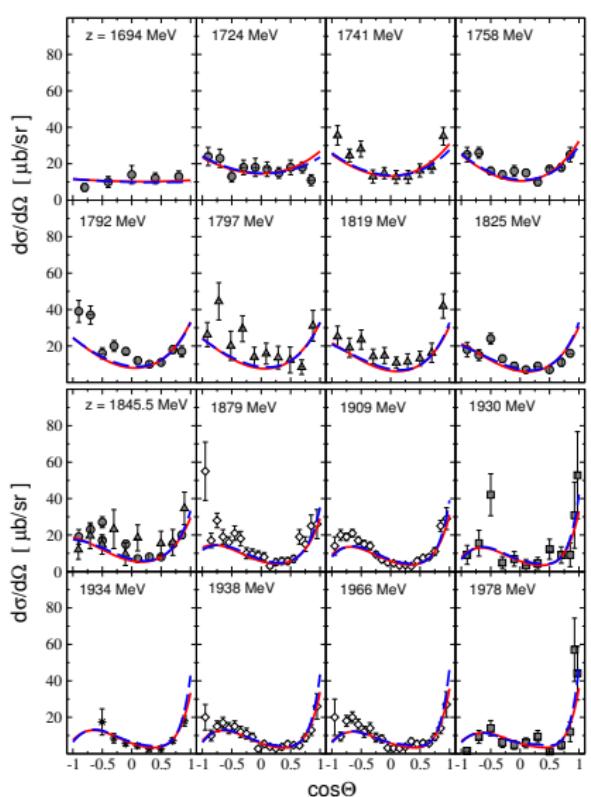
◀ back

$K\Lambda$: Partial wave content

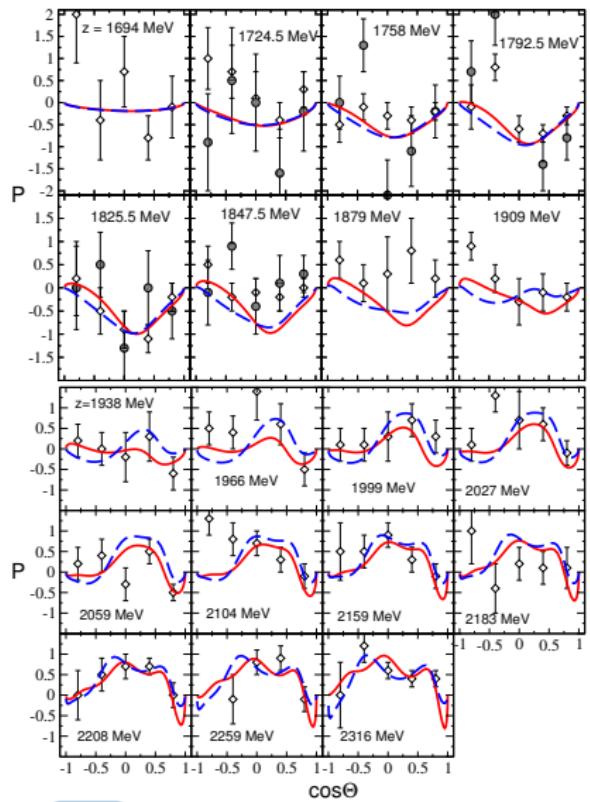


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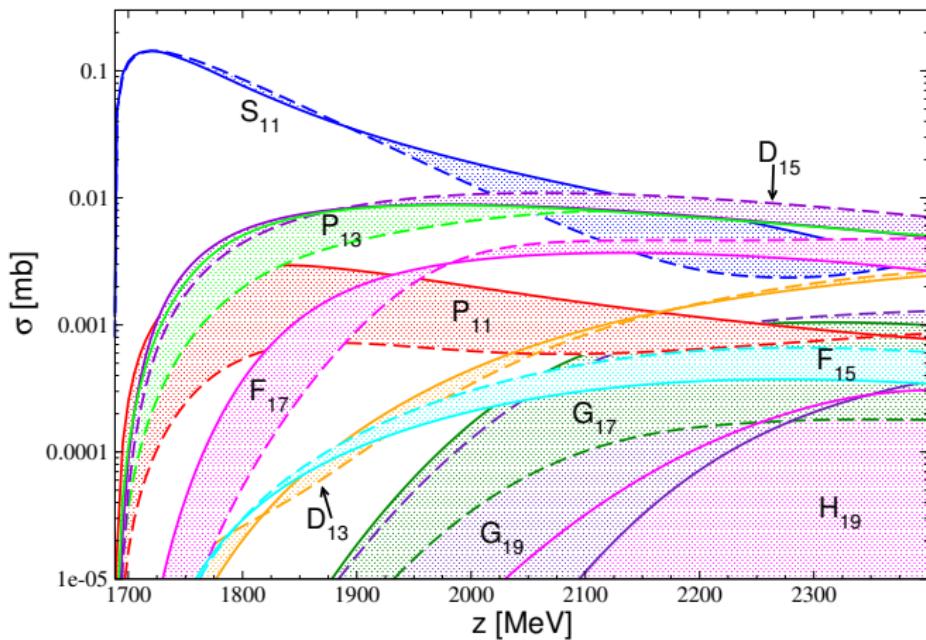
Full results $K^0\Sigma^0$: $d\sigma/d\Omega$

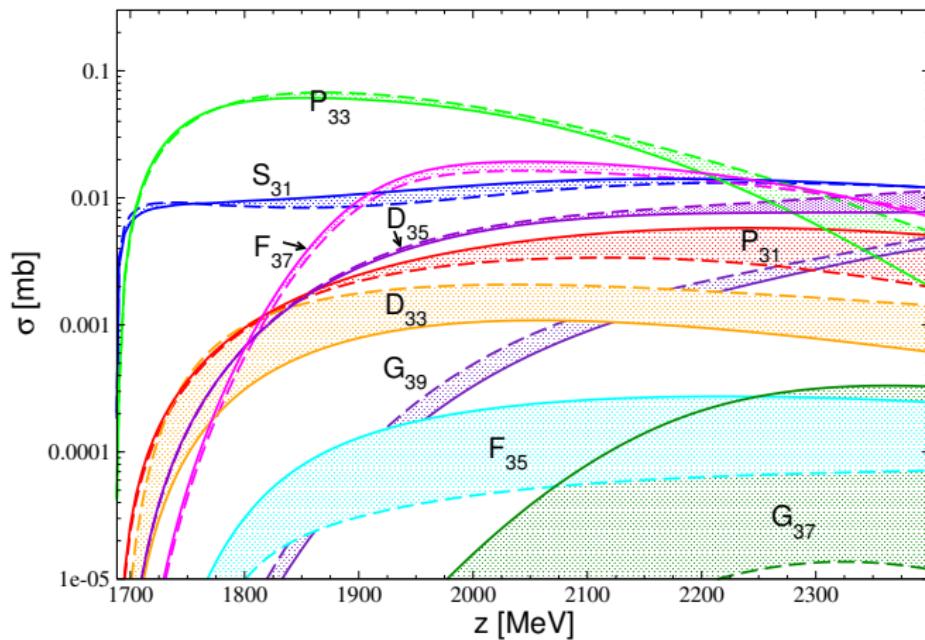


Full results $K^0\Sigma^0$: Polarization

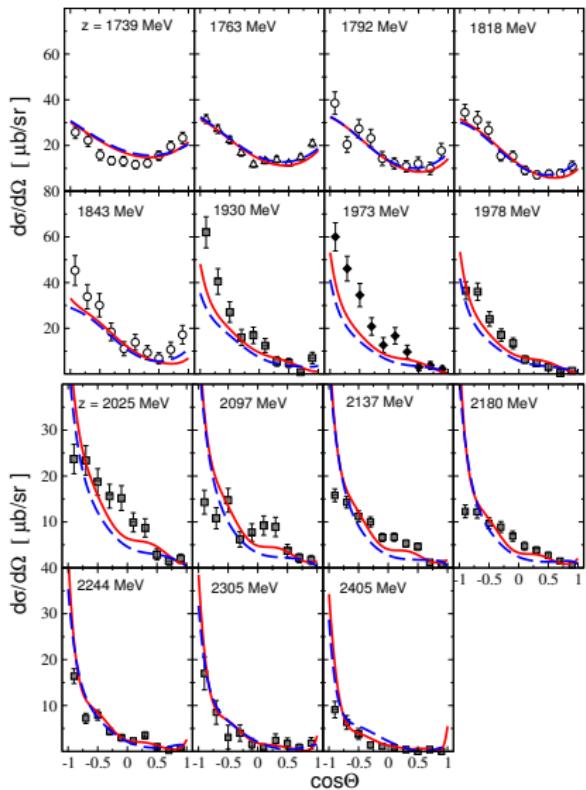


◀ back

$K^0\Sigma^0$: Partial wave content $l=1/2$ [◀ back](#)

$K^0\Sigma^0$: Partial wave content $l=3/2$ [◀ back](#)

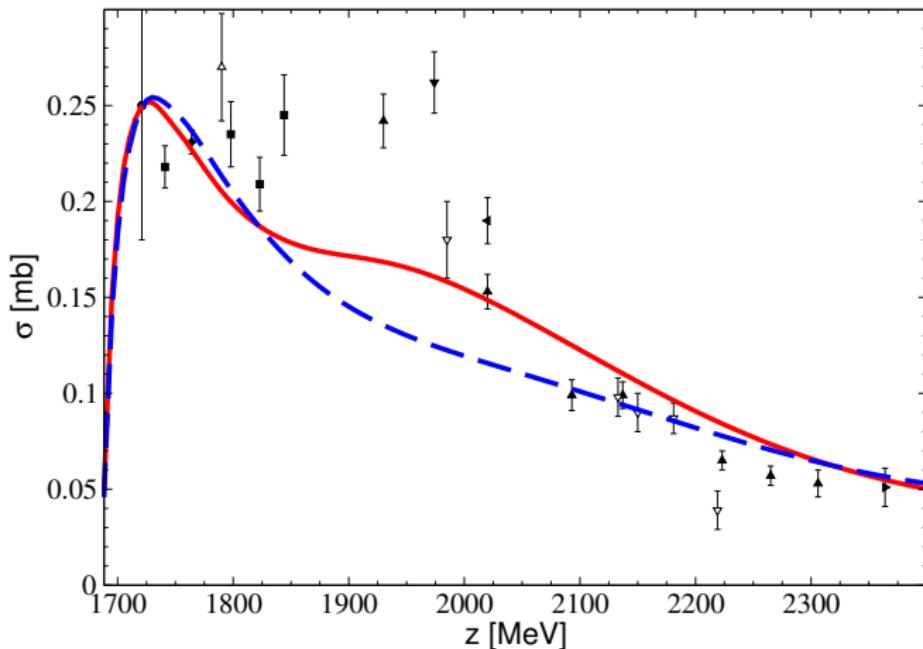
Full results $K^+\Sigma^-$: $d\sigma/d\Omega$



no Polarization data !

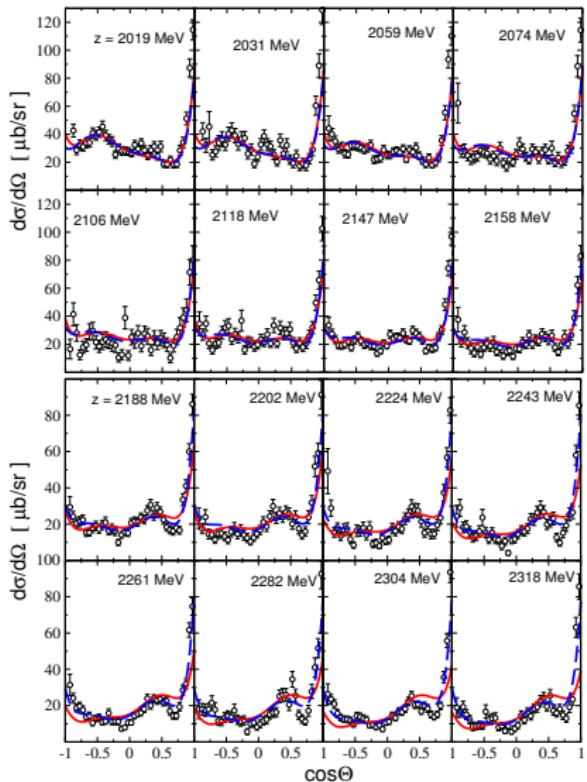
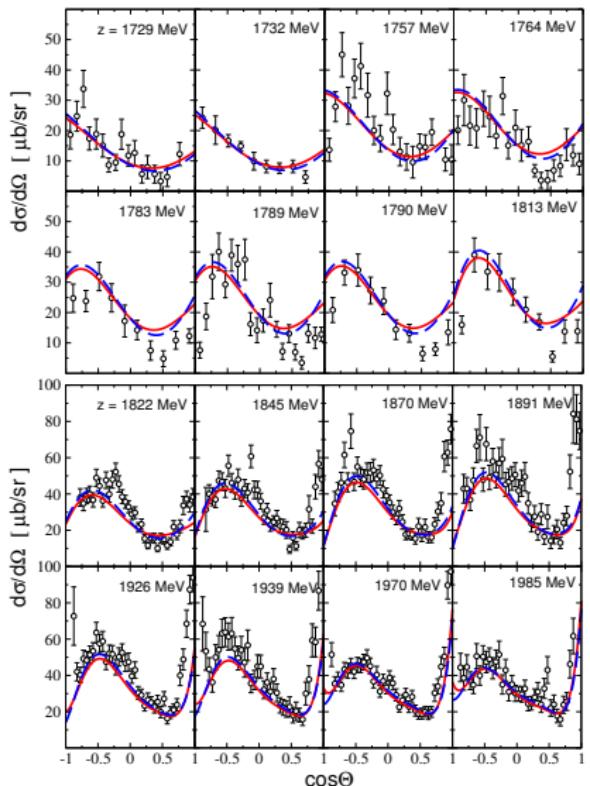
$\pi^- p \rightarrow K^+ \Sigma^-$: Total cross section

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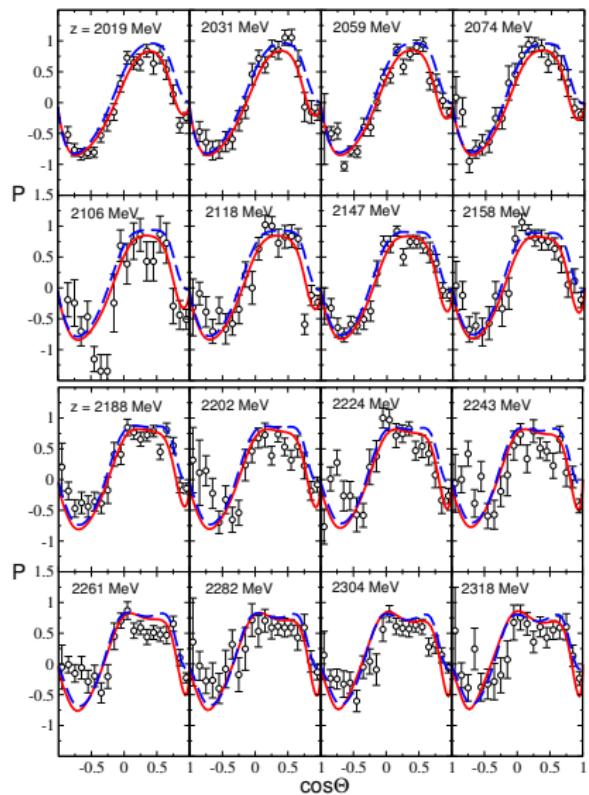
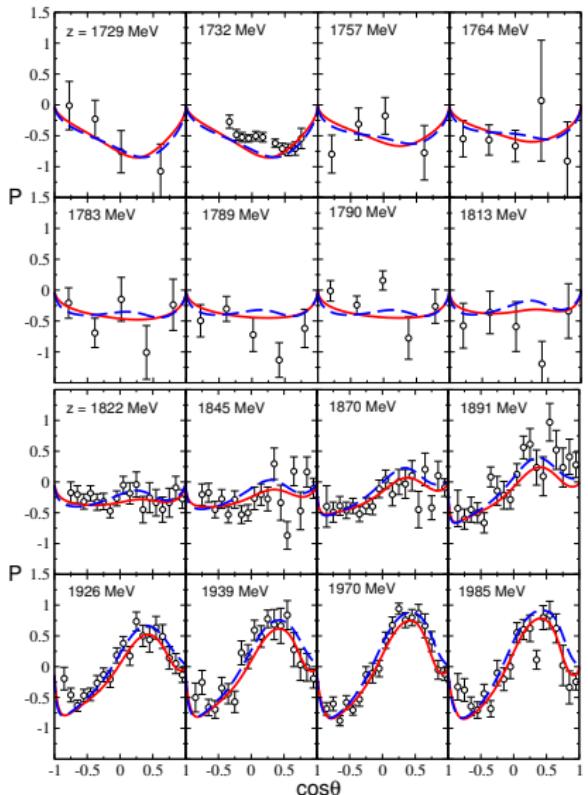
◀ back

Full results $K^+ \Sigma^+$: $d\sigma/d\Omega$



[◀ back](#)

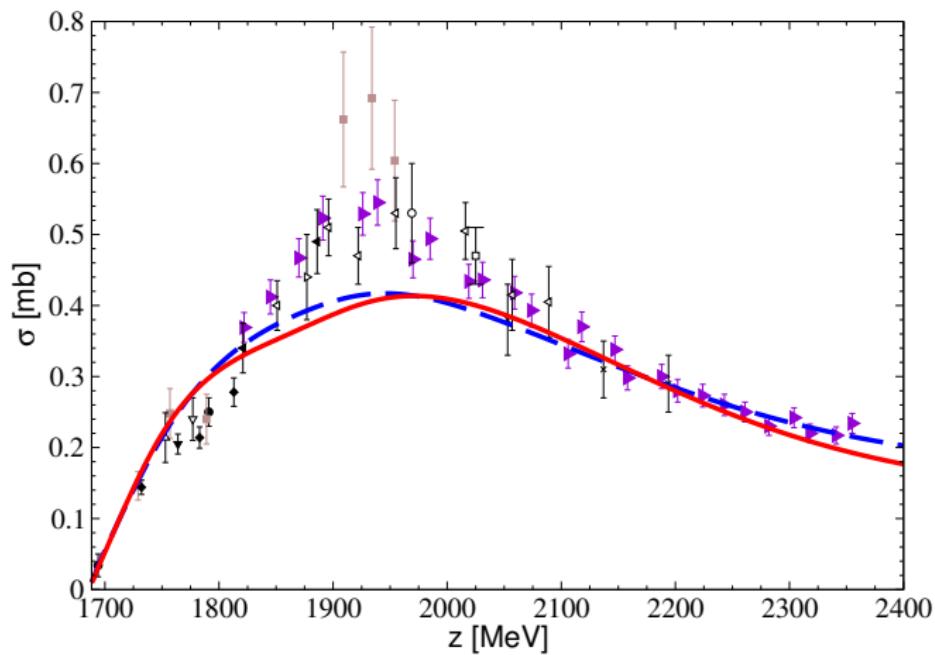
Full results $K^+\Sigma^+$: Polarization



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$\pi^+ p \rightarrow K^+ \Sigma^+$: Total cross section

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Resonance content: $I = 1/2$

| fit→ | Re z_0 | | -2Im z_0 | | $r_{\pi N}$ | | $\theta_{\pi N \rightarrow \pi N}$ | | $\Gamma_{\pi N}/\Gamma_{\text{tot}}$ | |
|--------------------------|----------|------|------------|-----|-------------|----|------------------------------------|------|--------------------------------------|------|
| | [MeV] | | [MeV] | | [MeV] | | [deg] | | [%] | |
| | A | B | A | B | A | B | A | B | A | B |
| N(1535) 1/2 $^-$ | 1498 | 1497 | 74 | 66 | 17 | 13 | -37 | -43 | 40 | 44 |
| N(1650) 1/2 $^-$ | 1677 | 1675 | 146 | 131 | 45 | 27 | -43 | -38 | 61 | 41 |
| N(1440) 1/2 $^+_{(a)}$ | 1353 | 1348 | 212 | 238 | 59 | 62 | -103 | -111 | 56 | 52 |
| N(1710) 1/2 $^+$ | 1637 | 1653 | 97 | 112 | 4 | 8 | -30 | 34 | 8.2 | 14 |
| N(1750) 1/2 $^+_{(*,a)}$ | 1742 | (nf) | 318 | - | 8 | - | 161 | - | 5.1 | - |
| N(1720) 3/2 $^+$ | 1717 | 1734 | 208 | 306 | 7 | 18 | -76 | -23 | 6.6 | 11.6 |
| N(1520) 3/2 $^-$ | 1519 | 1525 | 110 | 104 | 42 | 36 | -16 | 10 | 79 | 69 |
| N(1675) 5/2 $^-$ | 1650 | 1640 | 126 | 178 | 24 | 34 | -19 | -32 | 39 | 38 |
| N(1680) 5/2 $^+$ | 1666 | 1667 | 108 | 120 | 36 | 41 | -24 | -24 | 67 | 68 |
| N(1990) 7/2 $^+$ | 1788 | 1936 | 282 | 244 | 4 | 4 | -84 | -87 | 3.2 | 3.6 |
| N(2190) 7/2 $^-$ | 2092 | 2054 | 363 | 486 | 42 | 44 | -31 | -57 | 23 | 18 |
| N(2250) 9/2 $^-$ | 2141 | 2036 | 465 | 442 | 17 | 13 | -67 | -62 | 7.5 | 5.7 |
| N(2220) 9/2 $^+$ | 2196 | 2156 | 662 | 565 | 87 | 46 | -67 | -72 | 26 | 16 |

◀ back

Resonance content: $I = 1/2$ Branching ratios

| | $\frac{\Gamma_{\pi N}^{1/2} \Gamma_{\eta N}^{1/2}}{\Gamma_{\text{tot}}}$ | | | | $\theta_{\pi N \rightarrow \eta N}$ | | | | $\frac{\Gamma_{\pi N}^{1/2} \Gamma_{K\Lambda}^{1/2}}{\Gamma_{\text{tot}}}$ | | | | $\theta_{\pi N \rightarrow K\Lambda}$ | | | | $\frac{\Gamma_{\pi N}^{1/2} \Gamma_{K\Sigma}^{1/2}}{\Gamma_{\text{tot}}}$ | | | | $\theta_{\pi N \rightarrow K\Sigma}$ | | | |
|---------------------------|--|-----|-------|-----|-------------------------------------|-----|-------|------|--|-----|-------|------|---------------------------------------|---|-------|---|---|---|-------|---|--------------------------------------|--|--|--|
| | [%] | | [deg] | | [%] | | [deg] | | [%] | | [deg] | | [%] | | [deg] | | [%] | | [deg] | | | | | |
| | A | B | A | B | A | B | A | B | A | B | A | B | A | B | A | B | A | B | A | B | | | | |
| $N(1535) \ 1/2^-$ | 51 | 48 | 120 | 115 | 7.7 | 8.3 | 68 | 77 | 15 | 34 | -74 | -83 | | | | | | | | | | | | |
| $N(1650) \ 1/2^-$ | 15 | 12 | 57 | 46 | 25 | 18 | -46 | -43 | 26 | 15 | -63 | -59 | | | | | | | | | | | | |
| $N(1440) \ 1/2^+_{(a)}$ | 2 | 5 | -40 | -26 | 2 | 11 | 156 | 152 | 1 | 2 | 67 | 189 | | | | | | | | | | | | |
| $N(1710) \ 1/2^+$ | 24 | 23 | 130 | 164 | 9.4 | 17 | -83 | -41 | 3.9 | 0.1 | -136 | -112 | | | | | | | | | | | | |
| $N(1750) \ 1/2^+_{(*,a)}$ | 0.5 | - | -140 | - | 0.8 | - | -170 | - | 2.2 | - | 4 | - | | | | | | | | | | | | |
| $N(1720) \ 3/2^+$ | 1.2 | 3.1 | 98 | 117 | 3.1 | 2.9 | -89 | -63 | 1.7 | 2.2 | 64 | 90 | | | | | | | | | | | | |
| $N(1520) \ 3/2^-$ | 3.5 | 3.0 | 87 | 113 | 5.8 | 6.3 | 158 | 177 | 0.8 | 3.6 | 163 | 164 | | | | | | | | | | | | |
| $N(1675) \ 5/2^-$ | 6.0 | 3.6 | -40 | -66 | 0.3 | 1.7 | -93 | -122 | 3.3 | 3.7 | -168 | 179 | | | | | | | | | | | | |
| $N(1680) \ 5/2^+$ | 0.4 | 1.5 | -47 | -54 | 0.2 | 0.3 | -99 | 72 | 0.1 | 0.1 | 141 | 141 | | | | | | | | | | | | |
| $N(1990) \ 7/2^+$ | 0.4 | 0.5 | -99 | 90 | 1.7 | 1.5 | -123 | -99 | 0.8 | 1.2 | 28 | 70 | | | | | | | | | | | | |
| $N(2190) \ 7/2^-$ | 0.1 | 0.4 | -28 | 99 | 1.9 | 0.3 | -51 | -75 | 1.3 | 0.5 | -63 | -105 | | | | | | | | | | | | |
| $N(2250) \ 9/2^-$ | 0.6 | 0.1 | -92 | -96 | 1.1 | 0.7 | -103 | -106 | 0.3 | 0.7 | -114 | 62 | | | | | | | | | | | | |
| $N(2220) \ 9/2^+$ | 0.1 | 0.3 | 63 | 74 | 0.9 | 0.8 | 53 | 59 | 0.8 | 0.1 | -138 | 52 | | | | | | | | | | | | |

Resonance content: $I = 3/2$

| fit→ | Re z_0 [MeV] | | -2Im z_0 [MeV] | | $ r_{\pi N} $ [MeV] | | $\theta_{\pi N \rightarrow \pi N}$ [deg] | |
|---|-------------------|------|---------------------|-----|------------------------|----|---|-------|
| | A | | B | | A | | B | |
| | A | B | A | B | A | B | A | B |
| Δ(1620) 1/2 ⁻ | 1599 | 1596 | 71 | 80 | 17 | 18 | -107 | -107 |
| Δ(1910) 1/2 ⁺ | 1788 | 1848 | 575 | 376 | 56 | 20 | -140 | -143 |
| Δ(1232) 3/2 ⁺ | 1220 | 1218 | 86 | 96 | 44 | 50 | -35 | -38 |
| Δ(1600) 3/2 _(a) ⁺ | 1553 | 1623 | 352 | 284 | 20 | 27 | -158 | -124 |
| [Δ(1920) 3/2 ⁺ | 1724 | 1808 | 863 | 887 | 36 | 19 | 163 | -70] |
| Δ(1700) 3/2 ⁻ | 1675 | 1705 | 303 | 185 | 24 | 14 | -9 | -4 |
| Δ(1930) 5/2 ⁻ | 1775 | 1805 | 646 | 580 | 18 | 14 | -159 | 3 |
| Δ(1905) 5/2 ⁺ | 1770 | 1776 | 259 | 143 | 17 | 9 | -59 | -40 |
| Δ(1950) 7/2 ⁺ | 1884 | 1890 | 234 | 232 | 58 | 58 | -25 | -19 |
| Δ(2200) 7/2 ⁻ | 2147 | 2111 | 477 | 353 | 17 | 20 | -52 | 7 |
| Δ(2400) 9/2 ⁻ | 1969 | 1938 | 577 | 559 | 25 | 19 | -80 | -112 |

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Resonance content: $I = 3/2^-$ Branching ratios

| fit→ | $\Gamma_{\pi N}/\Gamma_{\text{tot}}$ | | $\frac{\Gamma_{\pi N}^{1/2}\Gamma_{K\Sigma}^{1/2}}{\Gamma_{\text{tot}}}$ | | $\theta_{\pi N \rightarrow K\Sigma}$ | |
|------------------------------------|--------------------------------------|-----|--|-----|--------------------------------------|------|
| | [%] | | A | B | A | B |
| $\Delta(1620) \, 1/2^-$ | 48 | 45 | 22 | 24 | -107 | -107 |
| $\Delta(1910) \, 1/2^+$ | 20 | 10 | 4.7 | 1.9 | -144 | -115 |
| $\Delta(1232) \, 3/2^+$ | 100 | 100 | | | | |
| $\Delta(1600) \, 3/2^+_{(\sigma)}$ | 11 | 19 | 11 | 13 | -7 | 41 |
| $[\Delta(1920) \, 3/2^+]$ | 8.3 | 4.3 | 16 | 14 | -20 | 50 |
| $\Delta(1700) \, 3/2^-$ | 16 | 16 | 1.5 | 1.6 | -150 | -121 |
| $\Delta(1930) \, 5/2^-$ | 5.6 | 4.8 | 3.1 | 1.7 | -3 | 135 |
| $\Delta(1905) \, 5/2^+$ | 13 | 12 | 0.5 | 0.1 | -142 | -99 |
| $\Delta(1950) \, 7/2^+$ | 50 | 50 | 4.0 | 3.8 | -78 | -71 |
| $\Delta(2200) \, 7/2^-$ | 7.2 | 11 | 0.6 | 0.1 | -98 | -33 |
| $\Delta(2400) \, 9/2^-$ | 8.7 | 5.7 | 1.3 | 0.8 | 40 | 6 |

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Data base

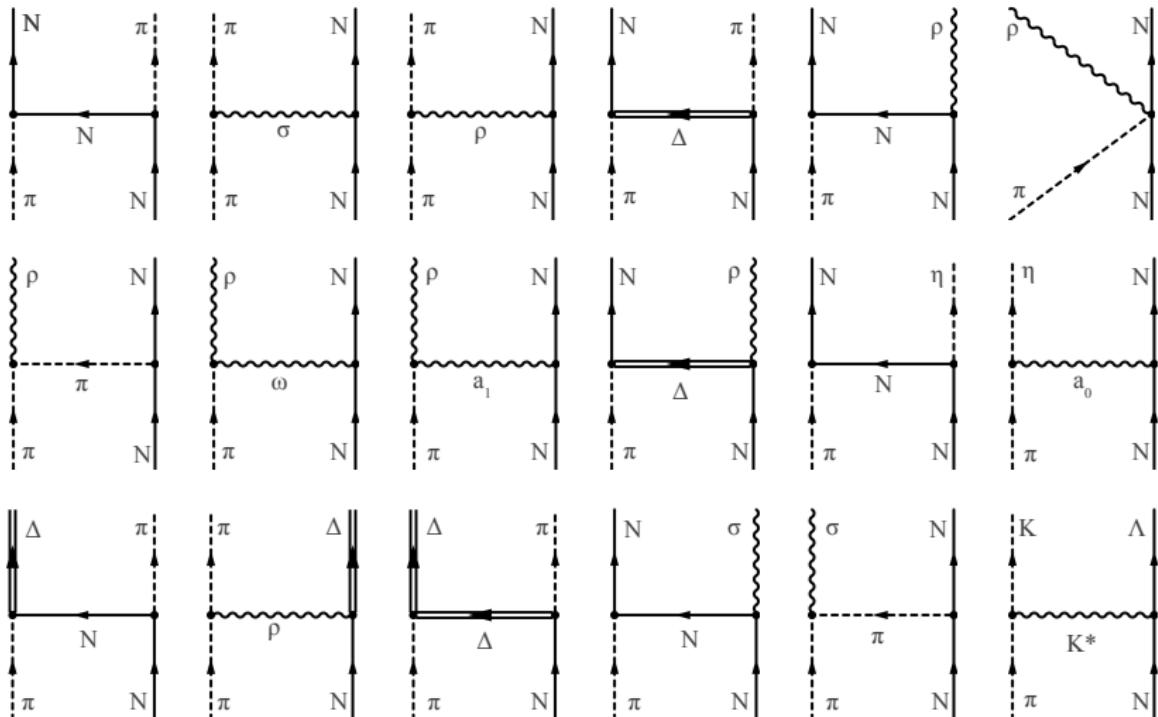
simultaneous fit to $\pi N \rightarrow \pi N, \eta N, K\Lambda, K\Sigma$

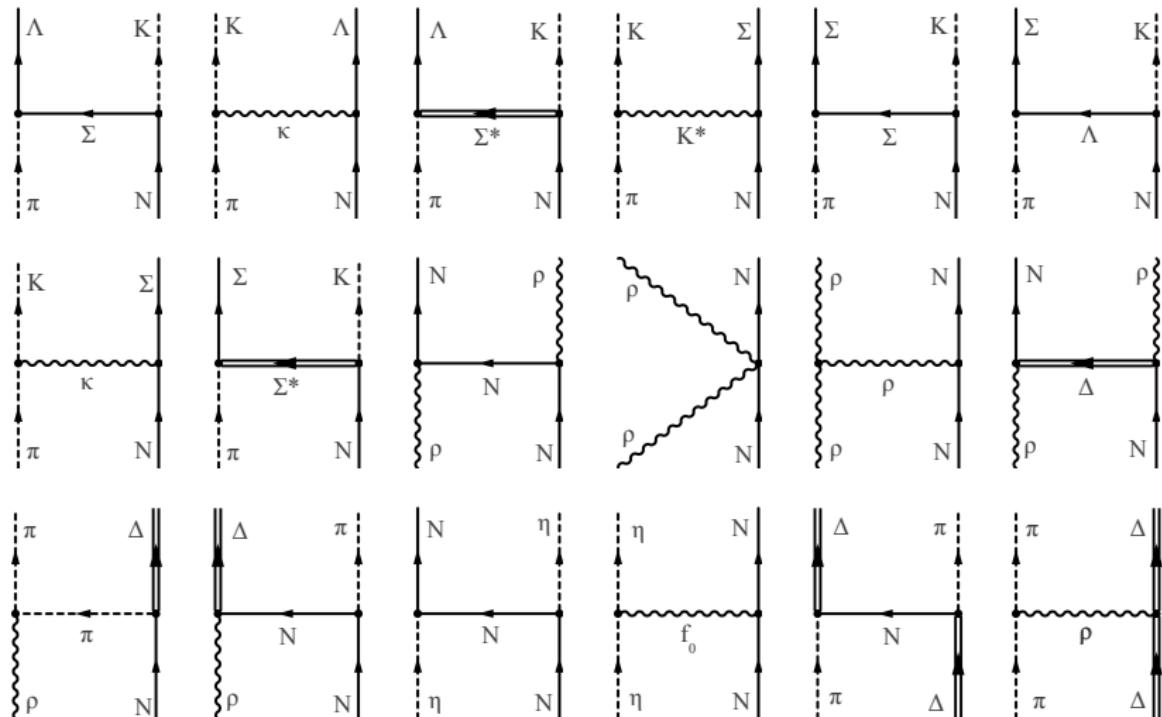
World data base on $\eta N, K\Lambda, K\Sigma$

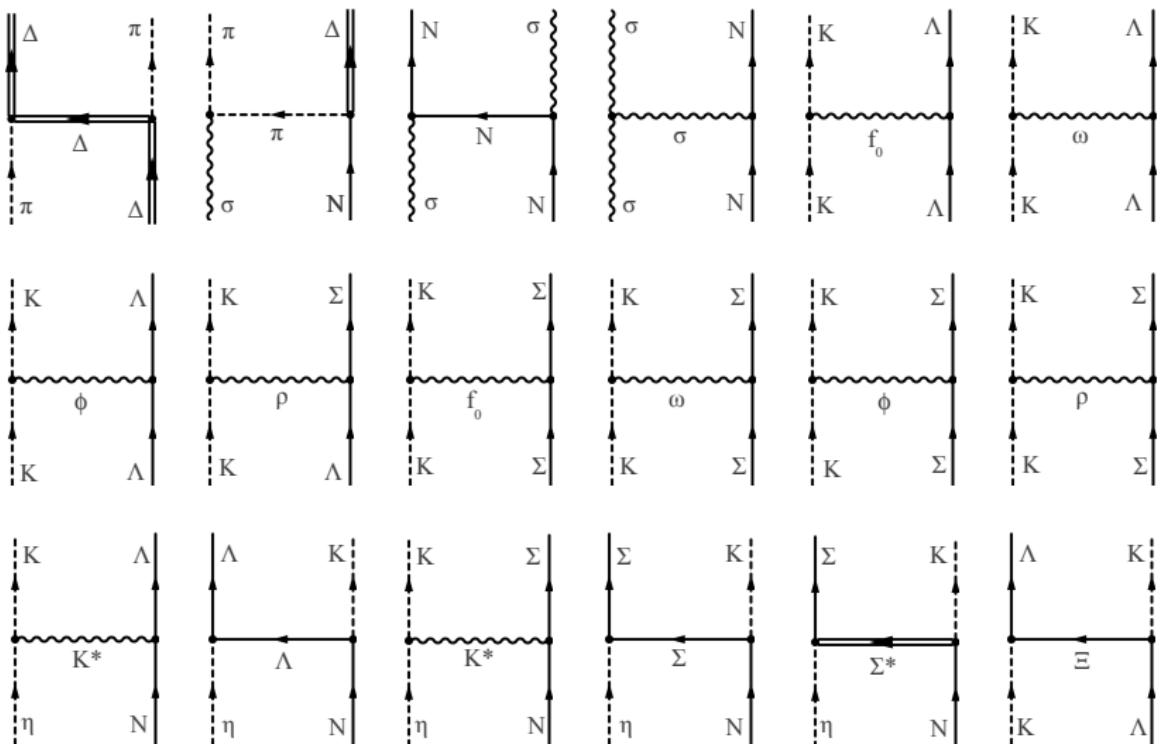
| | PWA | σ_{tot} | $\frac{d\sigma}{d\Omega}$ | P | β |
|------------------------------------|------------------------------|----------------|--|--------------------------------------|--------------------------------------|
| $\pi N \rightarrow \pi N$ | GWU/SAID 2006 up to J=9/2 | | | | |
| $\pi^- p \rightarrow \eta n$ | | 62 data points | 38 energy points $z=1489$ to 2235 MeV | 12 energy points 1740 to 2235 MeV | |
| $\pi^- p \rightarrow K^0 \Lambda$ | | 66 data points | 46 energy points 1626 to 1405 MeV | 27 energy points 1633 to 2208 MeV | 7 energy points 1852 to 2262 MeV |
| $\pi^- p \rightarrow K^0 \Sigma^0$ | | 16 data points | 29 energy points 1694 to 2405 MeV | 19 energy points 1694 to 2316 MeV | |
| $\pi^- p \rightarrow K^+ \Sigma^-$ | | 14 data points | 15 energy points 1739 to 2405 MeV | | |
| $\pi^+ p \rightarrow K^+ \Sigma^+$ | | 18 data points | 32 energy points 1729 to 2318 MeV | 32 energy points 1729 to 2318 MeV | 2 energy points 2021 and 2107 MeV |

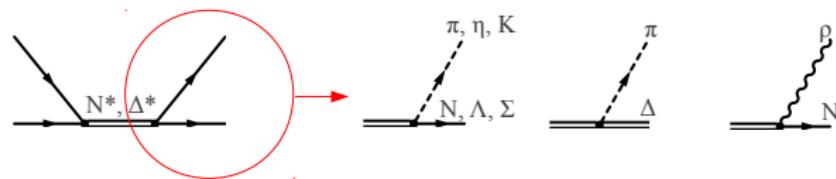
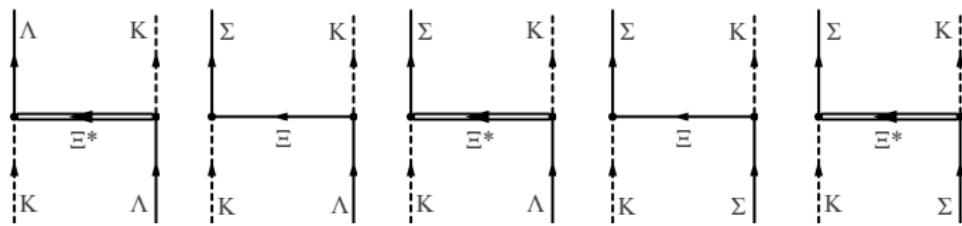
~ 6000 data points

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t and *u*, contd.; *s*-channel exchanges

Potentials for resonances with $J > 3/2$

- Correct dependence on the orbital angular momentum (centrifugal barrier)

$$v_{\gamma \frac{5}{2}^-} = \frac{k}{M} v_{\gamma \frac{3}{2}^+}$$

$$v_{\gamma \frac{7}{2}^-} = \frac{k^2}{M^2} v_{\gamma \frac{3}{2}^-}$$

$$v_{\gamma \frac{5}{2}^+} = \frac{k}{M} v_{\gamma \frac{3}{2}^-}$$

$$v_{\gamma \frac{7}{2}^+} = \frac{k^2}{M^2} v_{\gamma \frac{3}{2}^+}$$

$\gamma = \pi N, \rho N, \eta N, \pi \Delta, \sigma N, K \Lambda$ and $K \Sigma$

M : mass of the baryon in the particular channel

Phenomenological potential:

$$V_{\mu\gamma}(E, q) = \text{Diagram 1} + \text{Diagram 2} = \frac{\tilde{\gamma}_{\mu;i}^a(q)}{m_N} P_\mu^{\text{NP}}(E) + \sum_i \frac{\gamma_{\mu;i}^a(q) P_i^{\text{P}}(E)}{E - m_i^b}$$

Diagram 1: A nucleon (N) interacts with a photon (γ) via a phenomenological potential P_u^{NP} to produce a pion (π) and a nucleon (B). The pion has mass m .
Diagram 2: A nucleon (N) interacts with a photon (γ) via a phenomenological potential P_i^{P} to produce a resonance (N^*, Δ^*) and a nucleon (B). The resonance has mass m . The hadronic vertex $\tilde{\gamma}_{\mu;i}^a(q)$ is shown.

$P_i^{\text{P}}, P_\mu^{\text{NP}}$: energy-dependent polynomials

$\tilde{\gamma}_{\mu;i}^a, \gamma_{\mu;i}^a$: hadronic vertices \rightarrow correct threshold behaviour

i : resonance number per multipole

μ : channels $\pi N, \eta N, \pi \Delta$

Polynomials

Helicity multipoles $A_{L\pm}^h$:

- $J = L + 1/2$:

$$A_{L+}^{1/2} = -\frac{1}{2} [(L+2)E_{L+} + LM_{L+}]$$

$$A_{L+}^{3/2} = \frac{1}{2}\sqrt{L(L+2)} [E_{L+} - M_{L+}]$$

- $J = L - 1/2$:

$$A_{L-}^{1/2} = -\frac{1}{2} [(L-1)E_{L-} - (L+1)M_{L-}]$$

$$A_{L-}^{3/2} = -\frac{1}{2}\sqrt{(L-1)(L+1)} [E_{L-} + M_{L-}]$$

Matching to lattice

Prediction & analysis of lattice data

[M. Döring et al., EPJ A47, 163 (2011)]

Scattering equation:

$$T(q'', q') = V(q'', q') + \int_0^\infty dq q^2 V(q'', q) \frac{1}{z - E_1(q) - E_2(q) + i\epsilon} T(q, q')$$

Discretization of momenta in the scattering equation:

$$\int \frac{\vec{d}^3 q}{(2\pi)^3} f(|\vec{q}|^2) \rightarrow \frac{1}{L^3} \sum_{\vec{n}_i} f(|\vec{q}_i|^2), \quad \vec{q}_i = \frac{2\pi}{L} \vec{n}_i, \quad \vec{n}_i \in \mathbb{Z}^3$$

$$T(q'', q') = V(q'', q') + \frac{2\pi^2}{L^3} \sum_{i=0}^{\infty} \vartheta(i) V(q'', q_i) \frac{1}{z - E_1(q_i) - E_2(q_i)} T(q_i, q'),$$

$\vartheta^{(P)}(i)$ series

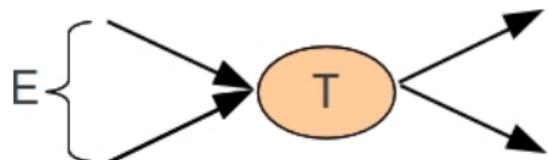
- Study finite size effects
- Predict lattice spectra

Two-body scattering

in the infinite volume limit

- Unitarity of the scattering matrix S : $SS^\dagger = \mathbb{1}$ $[S = \mathbb{1} - i \frac{p}{4\pi E} T]$.

$$\text{Im } T^{-1}(E) = \sigma \equiv \frac{p}{8\pi E}$$



- Generic (Lippman-Schwinger) equation for unitarizing the T -matrix:

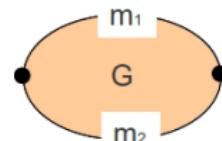
$$T = V + V G T \quad \text{Im } G = -\sigma$$

V : (Pseudo)potential, σ : phase space.

- G : Green's function:

$$G = \int \frac{d^3 \vec{q}}{(2\pi)^3} \frac{f(|\vec{q}|)}{E^2 - (\omega_1 + \omega_2)^2 + i\epsilon},$$

$$\omega_{1,2}^2 = m_{1,2}^2 + \vec{q}^2$$



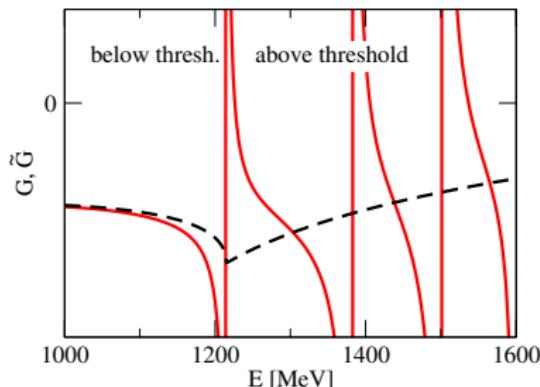
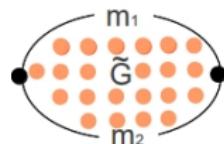
Discretization

Discretized momenta in the finite volume with periodic boundary conditions

$$\Psi(\vec{x}) \stackrel{!}{=} \Psi(\vec{x} + \hat{\mathbf{e}}_i L) = \exp(i L q_i) \Psi(\vec{x}) \implies q_i = \frac{2\pi}{L} n_i, \quad n_i \in \mathbb{Z}, \quad i = 1, 2, 3$$

$$\int \frac{d^3 \vec{q}}{(2\pi)^3} g(|\vec{q}|^2) \rightarrow \frac{1}{L^3} \sum_{\vec{n}} g(|\vec{q}|^2), \quad \vec{q} = \frac{2\pi}{L} \vec{n}, \quad \vec{n} \in \mathbb{Z}^3$$

$$G \rightarrow \tilde{G} = \frac{1}{L^3} \sum_{\vec{q}} \frac{f(|\vec{q}|)}{E^2 - (\omega_1 + \omega_2)^2}$$



- $E > m_1 + m_2$: \tilde{G} has poles at free energies in the box, $E = \omega_1 + \omega_2$
- $E < m_1 + m_2$: $\tilde{G} \rightarrow G$ exponentially with L (regular summation theorem).

Finite → infinite volume: the Lüscher equation

Warning: Very crude re-derivation.

- Measured eigenvalues of the Hamiltonian (tower of *lattice levels* $E(L)$)
→ Poles of scattering equation \tilde{T} in the finite volume → determines V :

$$\tilde{T} = (1 - V \tilde{G})^{-1} V \rightarrow V^{-1} - \tilde{G} \stackrel{!}{=} 0 \rightarrow V^{-1} = \tilde{G}$$

- The interaction V determines the T -matrix in the infinite volume limit:

$$T = (V^{-1} - G)^{-1} = (\tilde{G} - G)^{-1}$$

- Re-derivation of Lüscher's equation (T determines the phase shift δ):

$$p \cot \delta(p) = -8\pi\sqrt{s} (\tilde{G}(E) - \text{Re } G(E))$$

- V and dependence on renormalization have disappeared (!)
- p : c.m. momentum
- E : scattering energy
- $\tilde{G} - \text{Re } G$: known kinematical function
($\simeq Z_{00}$ up to exponentially suppressed contributions)
- One phase at one energy.**